

# Entertaining Malthus: Bread, Circuses and Economic Growth<sup>☆</sup>

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## Abstract

Motivated by the basic adage that man does not live by bread alone, we offer a theory of historical economic growth and population dynamics where human beings need food to survive, but enjoy other things, too. Our model imposes a Malthusian constraint on food, but introduces a second good to the analysis that affects living standards without affecting population growth. We show that technological change does a good job explaining historical consumption patterns and population dynamics, including the Neolithic Revolution, the Industrial Revolution, and the Great Divergence. Our theory stands in contrast to models that assume a single composite good and a Malthusian constraint. Such models are designed to match the belief that variations in living standards prior to the Industrial Revolution were negligible. Recent revisions to historical data show that historical living standards - though obviously much lower than today's - varied over time and space much more than previously thought. These revisions include updates to Maddison's data set, which served as the basis for many papers taking long-run stagnation as a point of departure. This new evidence suggests that the assumption of long-run stagnation is problematic. Our model shows that when we give theoretical accounting of these new observations the Industrial Revolution is much less puzzling.

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## 1. Introduction

A traditional view of historical economic growth is that prior to the Industrial Revolution there was very little change in per capita consumption.<sup>5</sup> This characterization is based largely on data from many studies summarized and augmented in Maddison (1982), which shows that median food consumption has generally been flat. The typical explanation is that humanity was stuck in a Malthusian trap. Roughly, this means that for per capita income above some subsistence level, population increased and below that level, it declined. Consequently, the population converged to a steady state at subsistence. In contrast, since the Industrial Revolution, countries with low per capita income have had high rates of population growth and countries with high per capita income have had much lower (or even declining) rates of population growth. At the same time, per capita consumption has soared.

This “hockey stick” characterization of historical consumption is seen as a puzzle and has led to the development of theories that can accommodate long-run stagnation followed by explosive and then sustained economic growth. The vast majority of these models assume a single composite good subject to a Malthusian constraint prior to the Industrial Revolution, which implies that consumption is kept at subsistence and growth is flat. Models vary however in how they explain explosive growth thereafter. After the Industrial Revolution, researchers generally assert, something fundamental must have changed. Hundreds of papers (we will review many of them in Section 2) have been written on this subject, each pointing to a different “something.” Perhaps the best-known example is unified growth theory, which explains the transition from long-run stagnation to explosive growth within a single framework and as part of the equilibrium path (Galor and Weil, 1999; Jones, 2001; Galor and Moav, 2002; Hansen and Prescott, 2002). For example, Galor and Moav (2002) posit that a single composite good consumed at subsistence due to a Malthusian trap leads to evolutionary pressures where survival favors individuals with abilities that accelerate technological progress. This leads to a state of technology requiring high human capital, which drives down fertility rates.

Over the past 10-15 years, new data coupled with new analyses of existing data have presented a more complicated picture of economic growth prior to the Industrial Revolution (see, for instance, Fouquet and Broadberry (2015) for a recent contribution). Perhaps most striking have been revisions to data from Maddison (1982), which rely less on median food consumption to draw conclusions about historical growth and consumption patterns. We will elaborate on the historical record below, but the new data on per capita GDP in the earlier era show a rich and interesting history rather than the history of long-term stagnation suggested by data on median food consumption. This

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<sup>5</sup>For example, Jones (2001) writes, “For thousands of years, the average standard of living seems to have risen very little, despite increases in the level of technology and large increases in the level of the population.” Or for example Hansen and Prescott (2002), who begin with the assertion that “Prior to 1800 living standards in world economies were roughly constant over the very long run...” Or Galor and Weil (1999, 2000), who assert: “This Malthusian framework accurately characterized the evolution of population and output per capita for most of human history. For thousands of years, the standard of living was roughly constant, and it did not differ greatly across countries.”

calls into question whether it is appropriate to build theories on the assumption that humanity was subject to a one-good Malthusian constraint prior to the Industrial Revolution.

In this paper, we make the following contribution to growth theory in general and to the modeling of the Malthusian constraint in particular. We argue that it is not the assumption of a Malthusian constraint that is problematic. Rather, it is the assumption of a single composite good coupled with the Malthusian constraint that should be revisited. We start with the basic idea that humanity needs food to survive, but argue that this is a constraint that is as binding today as it was prior to the Industrial Revolution. On the other hand, the story of the Industrial Revolution is not a story about rising food consumption, but of explosive growth in the output and consumption of other goods that do not necessarily enhance survival, but that do enhance the quality of life.

Hence we are led to study a two sector model with two outputs: bread, by which we mean basic nutrients, and circuses, by which we mean all those things that make life worth living. In particular, we take the most restrictive version of the Malthusian mechanism, where food determines population and in the long-run steady state is fixed at subsistence. Next, we add a second, non-food good (circuses), which has no impact on population dynamics, evolutionary fitness or procreation. We allow technological change to enhance either the bread sector or the circus sector. In this manner, we create a theoretical framework where food production, which governs population, is divorced from the production of other goods, which affect living standards.

The two sector model generates conclusions about the impact of sector-specific shocks. A distinctive conclusion from our model is that technological change that primarily enhances the bread sector - the Neolithic Revolution being the most dramatic instance - will largely increase population, but can actually lead to a decline in living standards measured by per-capita circus production. In other words, our model accommodates the idea that a positive technology shock can leave everyone worse off. The first implication can be derived from a standard, single-good Malthusian model. However, long-run declines in living standards are not possible in the standard model since living standards simply return to subsistence following a shock, which is equal to their pre-shock level.

Our model also implies that technological change that primarily enhances the circus sector - Ancient Rome being a key example - will largely increase long-run per capita GDP, but leave population the same or even lower. In the standard Malthusian model, a technological change can increase per capita GDP only in the short-run and must increase population.

We validate the theory against a variety of historical incidents for which data are available. Our procedure is to first determine which sector the technological shock impacted and then evaluate whether subsequent demographic and economic changes match our model predictions. Besides the Neolithic and Industrial Revolutions, we examine the Song Dynasty in China, the Irish Potato Famines, the Black Death and the Great Divergence. These historical episodes are quite different, but are all consistent with our model. Notice, the aim is not to deny the possibility that some rises in historical living standards were effectively consumed by commensurate rises in population. That story is fully accommodated by our model and would occur if rises in bread productivity are not accompanied by sufficient rises in circus productivity. While episodes of this sort are well

documented, we also provide evidence that there were sustained rises in living standards prior to the Industrial Revolution that were not immediately followed by rises in population. We argue from that data that this is because circus productivity rose, but bread productivity did not. This latter situation appears common in modern times. Our study has a very simple message: in light of recent revisions to historical data, it appears that the latter situation was perhaps not common in ancient times - but not non-existent. Models of historical growth should therefore be designed to accommodate this possibility.

Of particular interest is our analysis of the Great Divergence because our model suggests a novel account. The Great Divergence refers to the worldwide growth in the late Medieval period after Columbus and preceding the Industrial Revolution. Growth rates of total GDP in Europe and China were similar during this period, but in Europe per capita incomes went up while in China they declined. Our explanation of this is relatively straightforward: the import of New World crops represented a significant improvement in bread technology in those places suitable to their cultivation. Maize was suitable for cultivation in China and had a big impact on Chinese agriculture. Not so in Europe. We know that circus technology improved during this period. Even if we assume that circus technology improved as much in China as in Europe, the fact that bread technology improved in China but not Europe leads to the conclusion that per capita income should have risen in Europe relative to China. Therefore, our theory of the Great Divergence is that China was dragged down by agricultural improvement. This conclusion is buttressed if we observe that China was not the only place in which New World crops could be easily cultivated. The potato turns out to be particularly well suited to Ireland. Hence, if the theory is correct, Ireland - which surely had access to the same or similar circus technologies as (nearby) England - should never-the-less have seen a much greater population increase together with a per capita income fall relative to England - dragged down as they were by the potato. This is exactly what happened.

A second point to emphasize is the prediction our model makes about the negative technological shock in the bread sector in Ireland due to the potato blight that led to a famine in the mid-1800s (sometimes known as the Great Famine). Our model predicts a big decline in population, which accords with well-known facts. The model also predicts a large increase in per capita GDP - measuring from before the blight to the new steady state - which is a less obvious consequence of a negative shock to the bread sector. As we document, it is indeed the case that during this period per capita GDP in Ireland increased much more rapidly than in England.

In the next section, we review the literature on historical economic growth. We highlight some papers that are similar to ours so that our key departures are clear. Next, we present the theoretical model. This leads to the basic hypotheses that improvements in bread technology increase population and lower income while increases in circus technology leave population unchanged and increase income (and conversely for technological regression). We then examine the key historical epochs for consistency with these hypotheses: the Neolithic Revolution, the Roman Empire the Black Death plague, the Great Divergence and the Industrial Revolution.

## 2. Previous Literature on Historical Growth

In this section, we discuss the relevant literature in economics on historical economic growth. To facilitate comparison across papers, we summarize them in Table B.3. For each paper, the table includes: assumptions on preferences and technology, historical evidence on the abrupt shift in growth patterns to which the paper appeals and main findings.

We organize this review of literature around our key departures from earlier work. A first set of papers model early growth using a single good subject to a Malthusian constraint. This assumption seems to be at odds with (i) new evidence of sustained high living standards in history and (ii) the fact that food consumption has not exploded after the Industrial Revolution. A second set of papers considers two sectors. Most allow non-food goods to affect population growth. Moreover, many two-good frameworks limit substitutability between the sectors, for example, only allowing non-food goods to affect utility once a minimum amount of food is consumed. In contrast, we study a two-good setting where only one good, bread, drives population growth. We do not impose that goods be non-substitutable below some level of consumption. As we will demonstrate, our model can generate sustained rises and declines in living standards (depending on the sequence of shocks) despite a binding Malthusian constraint.

### *2.1. Malthusian Traps in Single Composite Good Economies*

Table B.3 highlights how nearly all of the papers we review are motivated by the assumption that growth prior to 1760 was negligible.<sup>6</sup> To explain this approximation, many researchers assume there is a single composite good that is subject to a Malthusian constraint and that economic growth is driven by endogenous technological advancements. The biggest problem with the single composite good is that living standards and food consumption are inextricably linked. As a result, explaining how Malthusian stagnation could be followed by explosive economic growth requires an abrupt shift in economic fundamentals (preferences or production functions) after some kind of living standards threshold is reached. For example, Arifovic, Bullard, and Duffy (1997) present a model where, through an adaptive learning process, and beyond a certain level of accumulated human capital, an economy moves from a low-growth to a high-growth equilibrium. Other papers (for example: Becker and Barro (1988), Lucas (2002) and Razin and Ben-Zion (1975)) employ dynastic utility functions to examine how endogenous fertility patterns interact with changes in technology. In these latter papers, there is a fundamental shift in how children are valued that is used to explain how stagnation gave way to growth.

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<sup>6</sup>For example, Arifovic, Bullard, and Duffy (1997) cite Maddison (1982) to conclude “Prior to industrialization, all of today’s highly developed economies experienced very long periods, *epochs*, of relatively low and stagnant growth in per capita income...While these data are highly aggregated and necessarily involve some guesswork, few economists would question the picture they paint.” Goodfriend and McDermott (1995) make a similar assertion: “Maddison, presents population and per capita growth rates since 500 AD. Although the numbers are highly aggregated both across countries and over time and are obviously imprecise, they tell a dramatic story. For the thousand years following the fall of Rome, there was little net progress in population and none in per capita product.” Other sources that are often used as evidence of stagnant growth include McEvedy, Jones et al. (1978), Clark (1998), Clark (2005) and Parente and Prescott (1993), among others.

A related approach, sometimes known as unified growth theory, develops models that explain within a single framework Malthusian stagnation followed by explosive and then sustained growth. The idea is that the Malthusian trap itself affected population dynamics in ways that would eventually lead to an escape from the Malthusian trap. For example, Galor and Moav (2002) argue that Malthusian pressures led to the evolutionary selection of traits that would lead to higher economic growth. Like other authors, however, they make the assumption that prior to the Industrial Revolution, growth in standards of living (and differences in living standards across countries) were negligible. In our view, this assumption is at odds with ongoing revisions to historical data. This means that different theories should be developed that accommodate recent empirical work. Herein lies our contribution.

In a second type of research on historical growth, authors posit models in which growth prior to 1760 is not necessarily flat. A key example is in Acemoglu and Zilibotti (1997), who develop a model emphasizing the high variability of output during early stages of development. In their setup, output growth is slow, but also subject to randomness.<sup>7</sup> They explain this with a model where capital projects are few and subject to indivisibilities, which limits risk-spreading and encourages investment in safer projects that are less productive.

Authors positing models of pre-Industrial Revolution growth appear to have hit upon the inconsistency of some historical data (indeed, the data they use to motivate their own models) with Malthusian assumptions, and so they are left with the task of reconciling their models with the widespread view that growth was negligible. One illustrative example is Temin (2013), who provides a detailed and convincing account of high living standards in Ancient Rome. Never-the-less, rather than argue that his evidence contradicts the standard view of Malthusian bleakness until the Industrial Revolution, he instead argues that Ancient Roman living standards constituted a temporary aberration, that is, an exception to what was otherwise a widespread Malthusian trap. We disagree. Instead, the model we posit in Section 3 develops conditions under which a binding Malthusian constraint can co-exist with rises in living standards.

Another example is Baumol (1990). Like most other authors, he states his general acceptance that pre-1760 growth is flat. In other passages, however, he expresses some doubts regarding the common practice of relying on food production to infer growth patterns. This leads to a noticeable tension in the paper. The model in Baumol (1990) focuses on the allocation of entrepreneurial resources to explain growth differences, the aim being to explain great leaps in economic growth. However, the historical evidence he offers points to opportunities in eras and places - that are neither Great Britain nor post-Industrial Revolution - where great wealth could be gained. An example is the High Middle Ages, when a smaller and shorter Industrial Revolution occurred. After listing a number of technological improvements, including, for example, better woven woolen goods which surely raised utility, Baumol (1990) states, "In a period in which agriculture probably occupied some

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<sup>7</sup>Relatedly, Voigtländer and Voth (2006) emphasize randomness in explaining why the Industrial Revolution began in Great Britain versus, say, France.

90 percent of the population, the expansion of industry in the twelfth and thirteenth centuries could not by itself have created a major upheaval in living standards.” However, in an accompanying footnote, he writes, “But then, much the same was true of the first half century of ‘our’ Industrial Revolution,” by which he refers to the one beginning in the 18th century. Nonetheless, Baumol (1990) goes on to say: “[I]t has been deduced from what little we know of European gross domestic product per capita at the beginning of the eighteenth century that its average growth in the preceding six or seven centuries must have been modest, since if the poverty of the later time had represented substantial growth from eleventh-century living standards, much of the earlier population would surely have been condemned to starvation.” This of course is not true in a two sector model where living standards rise due to greater circus consumption and can be low without starvation.

## 2.2. *Two Sector Economies*

Several papers are similar to ours in studying a two-sector model. One example is Hansen and Prescott (2002), who assume two types of production: a Malthus technology that uses a fixed factor (land) and a Solow technology that does not. Their motivation for the Malthusian technology is that its output has a considerable share of food in it and therefore requires land while the Solow production function results in mostly non-farm output (or “factory” output) and therefore requires very little land. The Solow technology, which produces non-farm output, appears similar to our circus sector technology. The biggest difference here is that while they divide the sectors production-wise - land-using or not, we divide the sectors consumption-wise - bread and circus have different demographic effects. Essentially, there is only one consumption good in their economy. Their model, however, implicitly assumes that individuals consider food and non-food items to be substitutable reproductively on a one-to-one basis. Given that their population growth function depends on consumption in general and not just on consumption of food, the model violates the basic Malthusian premise of population growth being kept in check by the availability of food.<sup>8</sup>

In a related paper, Yang and Zhu (2013) allow the Solow technology to govern both non-agricultural production, which never uses land, and farm production, which does initially use land. After industry develops, however, capital replaces land in farm production. In Yang and Zhu (2013), however, it is total consumption, agricultural and non-agricultural, that drives population growth. Further, defining consumer preferences by requiring food consumption to constitute a specific amount of total income with the rest being spent on non-food items essentially makes non-food production the true driver of population growth.

Another set of papers that are similar to us consider two consumption sectors. In Voigtländer and Voth (2013b), high growth is achieved by a major shock to population, the Black Death, which raised wages and also induced demand for goods produced in cities. As a result, wages in cities rose, which attracted new citizens. As death rates in urban centers were higher, population growth

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<sup>8</sup>It should be noted that Hansen and Prescott (2002) is not the only paper we have discussed in which a model is proposed that is not really Malthusian. In Acemoglu and Zilibotti (1997) and Arifovic, Bullard, and Duffy (1997), land is not a factor of production, which strictly speaking may be seen as violating a key Malthusian assumption.

was kept in check. In a different paper offering a compelling explanation of economic responses to the Black Death, Voigtländer and Voth (2013a) argue that a new Malthusian equilibrium arose prior to the Industrial Revolution where population is lower and wealth is higher. This occurs since land abundance induced by the plague led to a shift towards land-intensive pastoral agricultural production in which women had a comparative advantage. Higher female employment raised the opportunity cost of marriage and child-bearing, which lowered population and increased wealth. In Voigtländer and Voth (2013b), a form of lexicographic preferences means that non-food goods are enjoyed only after subsistence is reached. However, population dynamics are driven both by food and non-food consumption, with increases in the latter raising mortality rates. In Voigtländer and Voth (2013a), minimum consumption of a composite good is effectively assumed (the marginal utility of consumption below a “basic needs” level is large and positive) and the only other good in the economy over which women have preferences is their offspring, which is also tied to survival. In other words, in both papers, all goods affect survival and substitution between goods is limited.<sup>9</sup>

Another similar paper is Davies (1994) who models beef as a commodity that brings higher utility per calorie than potatoes.<sup>10</sup> Here, the idea is to add a subsistence constraint to classical demand analysis to show the Giffen effect. In particular, if humans consume meat and potatoes and are above, but near subsistence (an assumption that violates Malthusian precepts), a rise in potato prices could lead individuals to fall below subsistence unless they replaced beef with more potatoes. Here, the subsistence constraint is at the individual level versus the population level, which means that like other papers substitutability between goods is limited. In another paper, Taylor and Brander (1998) model a food sector and an “other goods” sector, which includes *moai*, the mysterious monumental statues on Easter Island. Their focus is on steady state dynamics to explain how the population of Easter Island may have been wiped out by an “over-correction” of population growth. In their setup, only labor is used in the non-food sector. Also, the food good is not subject to a strict Malthusian constraint. Instead, higher per-capita food consumption is assumed to increase the rate of population growth.

More similar to us, Strulik and Weisdorf (2008), Strulik and Weisdorf (2009) and Weisdorf (2008) develop sophisticated theories on how industrial productivity growth could restrain fertility - and thus raise living standards under a Malthusian constraint - by making children relatively more expensive than manufactured goods. Strulik and Weisdorf (2008) is probably the closest paper to ours. They model two sectors where population growth depends solely on prices in the food sector. There are three key differences. First, the model in Strulik and Weisdorf (2008) uses learning by doing and studies balanced growth equilibria. In contrast, we focus on technology shocks. Therefore, the model in Strulik and Weisdorf (2008) has difficulty explaining why certain

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<sup>9</sup>Somewhat similarly, in Moav and Neeman (2012) the circus good is replaced with one that serves a function as conspicuous consumption and therefore improves evolutionary fitness at the individual level. In our setting, circuses play no such evolutionary role.

<sup>10</sup>Relatedly, Lipsey, Carlaw, and Bekar (2005) study a highly involved three-sector model that includes a public good sector funded through taxation.



areas of the world experienced technology shocks that raised living standards considerably and then collapsed for reasons that have nothing to do with demography. Still, Strulik and Weisdorf (2008) do give evidence that population growth is responsive to food shocks. A second difference is that our Malthusian model is more strict - we use level of food rather than price of food to affect population. Third, there is no demand for children in our model. In contrast, Strulik and Weisdorf (2008) assume that utility is linear in children and manufactured goods and that children determine the growth rate of population in a linear way. This would be analogous to our assuming that bread and circuses are perfect substitutes. Therefore, their conclusions are rather different. In particular, we show that introducing a non-food sector that enhances utility, but that does not affect population growth, is sufficient to deliver the patterns of economic growth observed both before and after the Industrial Revolution in the presence of (and despite) the binding constraint that humans need food to survive.

### 2.3. Maddison Data and Its Revisions

Nearly all of the papers we have discussed (and that are mentioned in greater detail in Table B.3) rely at least to some extent on data from Maddison (1982) to motivate their models explaining historical growth patterns. Recent historians have found a richer picture of which we shall look at some specific instances subsequently.

It is worth mentioning that the original data set was meant to be a work-in-progress. The following excerpt from Bolt and van Zanden (2013) indicates this point.

*In view of the new research that has been done, many of the pre-1820 estimates (and all the pre-1600 figures) had to be modified. Maddison was of course aware of this: his strategy was to produce numbers even if a solid basis for them did not always exist, expecting that scholars might disagree and do new work to show that he was wrong. In this way he induced many scholars to work on these themes and to try to quantify long-term economic development. This was a highly successful strategy, but not always understood and appreciated by his colleagues; thanks to his pioneering work and the many, many reactions to it, we can now present a much more detailed overview of long-term economic growth than when he started his project in the 1960s.*<sup>11</sup>

Educated guesstimates constitute a valuable approach, but they are vulnerable to the contamination of the Malthusian presumption. As pointed out in a footnote in Galor and Weil (2000), parts of the Maddison (1982) data dealing with the pre-Industrial Revolution era may have been imputed with a Malthusian framework in mind. In other words, data imputed by pre-supposing Malthusian stagnation is used to motivate and test models of Malthusian stagnation.<sup>12</sup>

Turning to the evidence, a number of authors have begun to revise data on historical consumption, an undertaking that, according to the quotation above, was originally intended. Many results

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<sup>11</sup>Broadberry et al. (2010) echo the point that Maddison data were in part based on “guesstimates” meant to induce further research.

<sup>12</sup>Maddison (1995), it should be noted, argues that data imputation in Maddison (1982) does not presuppose Malthusian stagnation.

from this undertaking have been collected in Fouquet and Broadberry (2015) and Bolt and van Zanden (2013). We list some key contributors in Table 1 along with a note on their evidence and their main findings. In fact, some are not even that recent. Finley (1965) noticed that there was early evidence of technological progress and improvements to available goods. More recently, Broadberry et al. (2010) and van Zanden and van Leeuwen (2012) find evidence of pre-Industrial persistent economic growth in England and Holland, respectively. This is reflected in several recent revisions to the pre-1820 estimates of per capita GDP. The revised data show a far from stagnant economic landscape. We collect these revisions (and compare them to Maddison’s original numbers) in Table 2. For instance, Broadberry et al. (2010) show British per capita GDP doubling between 1270 and 1700 and doubling again between 1700 and 1870. Further, van Zanden and van Leeuwen (2012) find that Dutch per capita GDP more than tripled between 1000 and 1500. Malanima (2011) finds Italian per capita GDP to be declining between 1400 and 1820 from a level significantly higher than was suggested in the initial Maddison data set. Beyond suggesting a revision of our understanding of economic performance of these regions prior to 1820, these new estimates also justifiably cast doubt on the other flat parts of the pre-1820 Maddison data set which are currently under scrutiny.

Table 1: Literature From History on Pre-Modern Growth

Paper	Evidence	Key Findings
Finley (1965)	Various	Some evidence of technological progress in the ancient world.
Broadberry et al (2010)	Own data set	Evidence of sustained and persistent (if slow) growth in England, including growth in consumption goods.
Malanima (2011)	Data on Northern Italian agricultural production and urbanization	Evidence that pre-Industrial Revolution growth in Northern and Central Italy was stagnant.
Alvarez-Nodal and Prados de la Escosura (2012)	Own data set	Evidence that pre-Industrial Revolution growth in Spain was stagnant.
van Zanden and van Leeuwen (2012)	Own data set	Evidence of pre-Industrial Revolution persistent growth in Holland.
Foquet and Broadberry (2015)	Several data sets	Reject the “received wisdom” that growth in Europe prior to the Industrial Revolution was stagnant.

Table 2: Original Maddison Data on Historical Consumption Compared to Recent Revisions (in bold)

Year	Italy	<i>N. Italy</i>	Holland	<i>Holland</i>	England	<i>England</i>	Spain	<i>Spain</i>
1	809	*	425	*	400	*	498	*
1000	450	*	425	*	400	*	450	*
1348	*	<b><i>1,486</i></b>	*	<b><i>876</i></b>	*	<b><i>919</i></b>	*	<b><i>907</i></b>
1400	*	<b><i>1,716</i></b>	*	<b><i>1,195</i></b>	*	<b><i>1,205</i></b>	*	<b><i>819</i></b>
1500	1,100	<b><i>1,503</i></b>	761	<b><i>1,454</i></b>	714	<b><i>1,134</i></b>	661	<b><i>846</i></b>
1600	1,100	<b><i>1,336</i></b>	1,381	<b><i>2,662</i></b>	974	<b><i>1,167</i></b>	853	<b><i>892</i></b>
1700	1,100	<b><i>1,447</i></b>	2,130	<b><i>2,105</i></b>	1,252	<b><i>1,540</i></b>	853	<b><i>814</i></b>
1800	*	<b><i>1,336</i></b>	*	<b><i>2,609</i></b>	*	<b><i>2,200</i></b>	*	<b><i>916</i></b>
1820	1,117	*	1,838	*	1,706	*	1,008	*

This table presents historical data originally from Maddison (2007) alongside more recently revised data. Following Maddison (2007), units are per capita GDP levels in 1990 Geary-Khamis dollars. Data from other papers were converted to the same units in Bolt and van Zanden (2013). For each country, the Maddison data (in regular type) are listed adjacent to the more recently revised data (in bold and italicized type). Notice that data are not always available for exactly the same years, in which case the cell has a star. Revised data sources are as follows: Malanima (2011) for North Italy, van Zanden and van Leeuwen (2012) for Holland, Broadberry et al. (2010) for England and Nogal and de la Escosura (2013) for Spain.

### 3. Model

We believe that the typical characterization of growth throughout history is misleading. Humans do indeed need food to survive and population is therefore bound by a biological constraint, but this was as true prior to 1760 as it is today. In other words, imposing a Malthusian constraint relating food supply and population growth is a reasonable approximation, but the constraint applies to modern economies, too. On the other hand, humans neither need nor want to consume much more food than is biologically necessary. Therefore, explosive growth in food consumption has never been and should never be an expected byproduct of economic growth. Instead, growth should be measured in light of consumption of other, non-food goods. Finally, imposing the same type of “Malthusian” constraint with regard to the consumption of all other goods does not make sense as most other goods are not necessary for survival, but do affect the quality of life.

Therefore, a better approximation of how consumption and demographic change interact throughout history is a model that captures our need for food along with our desire for everything else. To that end, we present a model that is simple, based on standard neoclassical economic theory and entirely capable of delivering the result that technology change can drive economic growth. The key innovation is to impose a Malthusian constraint that links food supply to population growth, and to also permit a second good that is enjoyable, but that does not affect demographics, either through a Malthusian constraint or through some other device like evolutionary fitness. Notice that, absent the assumption of Malthusian bleakness for most of human history, we are no longer compelled to assume something else that would produce an abrupt escape from it. Rather, we show that in spite of a Malthusian constraint that binds food to population growth, changes in technology are

sufficient for explaining growth patterns prior to 1760 along with growth patterns thereafter.

In particular, we examine a simple two-sector model. There are two goods, which we call bread  $X$  and circuses  $Y$ . These are produced using two inputs, land  $L$  and people  $N$ . The endowment of land is fixed at  $L$ , while at a particular moment of time the endowment of people (labor) is denoted by  $N$ . The number of people, however, will be endogenously determined by a Malthusian dynamic. We focus on the most standard formulation for technology: the Cobb-Douglas.

The technologies for bread and circus production are of the following form.

$$X = BL_B^{1-\gamma_B} N_B^{\gamma_B} = (\beta L_B)^{1-\gamma_B} N_B^{\gamma_B} \text{ where } \beta = B^{1/(1-\gamma_B)}$$

$$Y = CL_C^{1-\gamma_C} N_C^{\gamma_C} = (\xi L_C)^{1-\gamma_C} N_C^{\gamma_C} \text{ where } \xi = C^{1/(1-\gamma_C)}.$$

Here  $B, C$  represent overall productivity in the bread and circus sector and we introduce also  $\beta, \xi$  measuring productivity in terms of land productivity rather than overall productivity. Our assumption about the production technology is that output elasticity in the bread sector with respect to land is greater than that in the circus sector: that is, that  $\gamma_B \leq \gamma_C$ .

The economy is subject to standard resource constraints,

$$L_B + L_C \leq L, N_B + N_C \leq N.$$

There is a single representative consumer whose per capita consumption of the two goods is denoted by

$$x = X/N, y = Y/N$$

and has utility function,

$$U(x, y) = \frac{x^{1-\rho} - 1}{1-\rho} + \frac{y^{1-\sigma} - 1}{1-\sigma}$$

with  $\rho, \sigma > 0$ . In other words, man does not live by bread alone. This reduces to the CES utility function if  $\rho = \sigma$  but in order to allow bread to be an inferior good, we assume that  $\sigma \leq \rho$ . Our other primary assumption is that the benefit of circuses does not decline too much as consumption of circuses increases: we assume that  $\sigma < 1$  which in the CES case means that the utility function exhibits greater substitutability than the Cobb Douglas.<sup>13</sup>

### *Population Dynamics*

Economists do not see individuals as maximizing survival and the Malthusian model does not suppose that population grows above the level of subsistence and that everybody instantly dies the

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<sup>13</sup>Unlike Voigtländer and Voth (2013b) we do not use the Stone-Geary linear expenditure formulation with a subsistence level for food. While this does allow bread to be an inferior good it implies for fixed bread consumption that the expenditure share of circuses remains constant as circus consumption increases. As we discuss below, the evidence suggests this is not the case.

moment food consumption falls below subsistence. We consider a population dynamic in which lower food consumption leads to lower birth-rates and higher death-rates. Further, we assume that the trade-off between bread and circuses for an individual is the usual type of trade-off studied by economists. Specifically, for any given population level  $N$  we model the economy as a competitive market for bread and circuses. Equivalently, according to the first welfare theorem, we assume that labor is allocated between the two sectors to maximize the utility of the representative consumer.

Population is assumed to move according to the usual Malthusian dynamic with respect to food consumption. We use  $\bar{x}$  to denote the subsistence level of bread - circuses being pleasant but unnecessary for survival. Recalling that  $x$  is the per capita output of bread, it follows that population increases if  $x > \bar{x}$  and declines if  $x < \bar{x}$ . Hence we will be interested in the Malthusian long run steady state equilibrium population level where  $N = \frac{X}{\bar{x}}$ .<sup>14</sup>

### *3.1. Evidence for Model Assumptions*

Before discussing comparative statics, we present evidence supporting the key assumptions in our model. First, we discuss evidence in support of the assumption that food is bound by a Malthusian constraint. Second, we show evidence on the substitutability of bread and circuses even among groups who are arguably at or near subsistence.

#### *3.1.1. Food Consumption Near Subsistence*

Our model assumes that per capita food consumption remains at (or near) subsistence most of the time. Prior to the Industrial Revolution there is strong evidence this is true. Indeed, for a long time economic historians only had data on food consumption and found it was quite flat (the handle of the hockey stick). However: food consumption remained flat also after the Industrial Revolution. Below we report U.S. food consumption in the 20th century: it did not increase much until about 1980 (Figure 1).<sup>15</sup> The implication is that GDP includes many goods that do very little (in our model: nothing) to encourage survival, but do constitute economic growth. Economists do not see individuals as maximizing survival. Rather, agents trade off longevity with other consumables that improve the quality of life.

#### *3.1.2. Food and Circuses are Substitutes*

In our framework, we assume that bread and circuses are sufficiently substitutable. This is different from authors who assume, for example, that agents derive utility from non-food items only after a certain subsistence level has been reached. It is precisely this degree of substitutability that makes the economy substitute circuses for bread in the face of circus sector improvements making

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<sup>14</sup>Allowing technology to grow continuously would pull the average bread consumption slightly away from the steady state equilibrium. We abstract from this complication because introducing it would not qualitatively affect results.

<sup>15</sup>Using per capita food supply as the sole measure of societal wealth and well-being is also problematic since in the U.S. and other developed (and even developing) countries, obesity has been associated with poverty rather than affluence. See, for example, Miech et al. (2006).

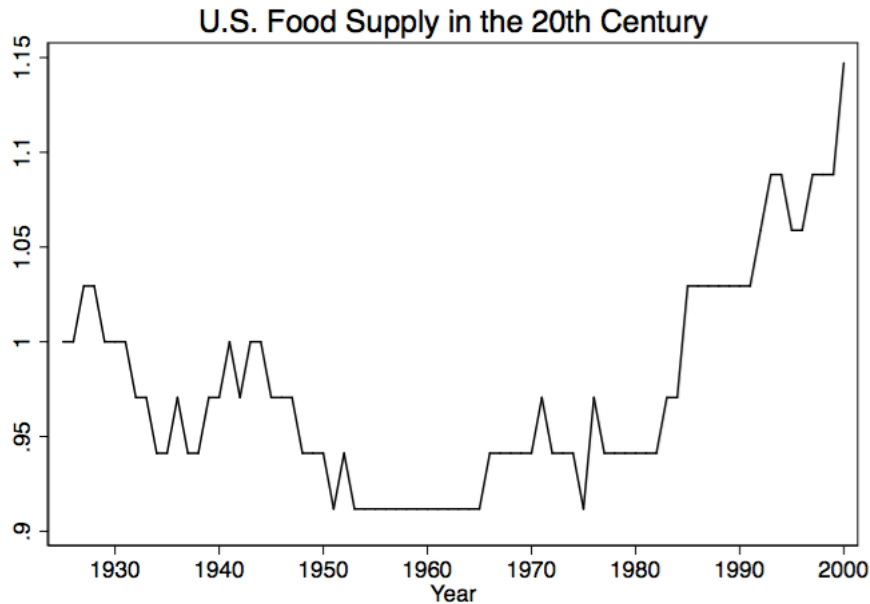


Figure 1: Calories per capita in the U.S. from 1909-2000.

larger populations unsustainable in equilibrium, leading to smaller economies with higher per capita GDP.

It turns out that such substitutability has considerable empirical support. Banerjee and Duflo (2011) extensively study consumption by very poor people in the modern era - those who live near subsistence. The authors document how poor people in the developing world spend a significant fraction of their income on weddings and funerals - and afterwards lament their lack of food. “In Udaipur, India, where almost no one has a television, the extremely poor spend 14 percent of their budget on festivals.” When television sets are available they are widely owned - and the poor people that own them maintain that television is more important than food. When people who are hungry are given a choice between more calories or better flavor, they often choose better flavor. These are scarcely new or controversial observations: much the same point is made by George Orwell in Orwell (1937). Related, in modern health economics, the value of a medical innovation is sometimes misleadingly computed using its effect on survival. Recent work has shown that this value should take into account that dynamically optimizing agents are willing to forfeit years of life when doing so can increase comfort or consumption.

### 3.2. The Malthusian Equilibrium and Comparative Statics

A *short run equilibrium* is the allocation of land and labor between the two sectors that maximizes utility for a given population  $N$ . A *Malthusian (or long-run steady state) equilibrium* is a short-run equilibrium where  $N$  is chosen so that consumption is at subsistence:  $x = \bar{x}$ . We now give the basic properties of equilibrium, proven in the Appendix.

**Proposition 1.** *For any  $N$  there is a unique short-run equilibrium. For any  $\bar{x}$  there is a unique Malthusian equilibrium.*

The following result is immediate and shows that our utility function captures the key fact that circus consumption and circus expenditure move in the same direction:

**Proposition 2.** *In a Malthusian equilibrium changes in technology that lead to higher per capita circus consumption  $y$  result in an increase in the share of expenditure on circuses  $yU_y(\bar{x}, y)/(\bar{x}U_x(\bar{x}, y)) = y^{1-\sigma}/\bar{x}^{1-\rho}$ .*

To make sensible comparisons across time we compute real per capita GDP by assigning fixed weights to bread and circus. Since in any Malthusian equilibrium, bread consumption is fixed at  $\bar{x}$ , increases or decreases in circus production/consumption are reflected by corresponding changes in real per capita GDP. Define *relative circus productivity*

$$\zeta = \frac{\xi}{\beta} = \frac{C^{\frac{1}{1-\gamma_C}}}{B^{\frac{1}{1-\gamma_B}}}$$

to be the relative land productivity in the circus sector relative to the bread sector. It is convenient to parameterize the economy by  $\beta, \zeta$ . In this case a technological improvement means a higher  $\beta$ . Such an improvement is bread-favoring if  $\zeta$  declines and circus-favoring if  $\zeta$  increases. Our main result is

**Proposition 3.** *Long run real per capita GDP  $y$  is a strictly increasing function of relative circus productivity  $\zeta$ .*

This says that technological change that increases circus productivity (measured in land units) more than bread productivity increases per capita GDP and conversely if bread productivity increases more than circus technology. This is very much at odds with standard Malthusian theory which says that long run per capita GDP should be independent of technological change.

The implications of the model for population are more subtle

**Proposition 4.** *Population is strictly increasing without bound in  $\beta$ . So, given an initial  $\beta$  and  $\zeta$  and a technology shock  $\zeta'$ , there exists  $\bar{\beta}$  such that  $N' > N$  iff  $\beta' > \bar{\beta}$ . Moreover,  $\bar{\beta} \leq \beta$  iff  $\zeta' \leq \zeta$ .*

In words, bread-favoring technological improvement lowers per capita income and raises population. Circus-favoring technological improvement raises per capita income and raises population if the improvement in bread technology is sufficiently great and lowers population if the improvement in bread technology is too small. With circus-favoring technological regression (for instance  $\beta$  declines with  $\xi$  fixed due to climate change or crop disease), per capita income goes up and population goes down.

A remark on our choice of technology parameterization is due. Relying on  $\beta$  and  $\zeta$  instead of  $B$  and  $C$  allows for the clean characterization of the relationship between technological changes and growth as captured by the propositions above. More importantly, we are interested in what

happens when technological change improves both  $B$  and  $C$ . We have the intuition that if  $C$  goes up fast enough relative to  $B$  then we should see an increase in per capita income. One way to look at this is to assess what happens if they go up in proportion. But there is nothing special about this particular benchmark and in fact we discovered that it is not a terribly helpful one. Instead, we use as our benchmark the ratio  $\zeta$  which allows us to obtain an exact characterization of what it means for  $C$  to increase enough relative to  $B$  that per capita income increases. Of course we want to know that when both  $C$  and  $B$  increase, we can get population to increase as well, but that is well identified by Proposition 4 which says that any technological change leads to increased population if it is big enough.

We also give one short-run result concerning exogenous decreases in the population:

**Proposition 5.** *If  $\gamma_B = \gamma_C$  then a decrease in  $N$  results in a short-run equilibrium with a higher wage (in bread units) and higher per capita GDP.*

By continuity the result also holds if  $\gamma_B$  is not too much smaller than  $\gamma_C$ . This shows that this model exhibits the same basic characteristics as a variety of other models used to analyze the consequences of shocks such as the Black Death Plague.

We can summarize the relevant comparative statics with the following hypotheses about the long-run

- A technological improvement that lowers relative circus productivity will increase population, decrease per capita GDP and lower expenditure share on circuses.
- A technological improvement that increases relative circus productivity will increase per capita GDP, increase expenditure share on circuses, will increase population if it is strong and decrease population if it is weak.
- A technological shock that lowers productivity in the bread sector will decrease population, raise per capita GDP and increase expenditure share on circuses.
- A shock that decreases population in the short run results in a higher wage and higher per capita GDP.

#### 4. Historical Evidence for Model Implications: Positive Technology Shocks

The model presented in the previous section generates predictions about the impact of shocks on demographics and consumption. We now discuss whether historical evidence supports model implications. We study historical epochs characterized by technological shifts and then examine subsequent demographic and economic transitions. The key to inspecting historical evidence in light of model predictions lies in our ability to determine the sector in which technology change occurred. The more precise predictions of the model map sector-specific shocks (or combinations of shocks) to shifts in population and living standards.



We discuss the following historical epochs: the Neolithic Revolution, the Classical Period (including Ancient Rome and Song China) the Late Medieval period, in particular, the Great Divergence and then the Industrial Revolution. In the following section, we also discuss “negative” shocks, including the Black Death and two potato famines in Ireland, episodes which we argue are also accommodated by our model. The historical examples we refer to are chosen because they are periods characterized by large technology shocks that led to substantial demographic and economic transition, which means it is possible to observe changes many centuries later.<sup>16</sup> To organize our discussion, for each epoch, we first discuss evidence that identifies the type of shock (or combination of shocks). Second, we refer to model hypotheses about population and consumption given the type of shocks. Third, we discuss whether data support model implications.

#### *4.1. Neolithic Revolution*

Perhaps the best historical example of an improvement to bread technology without a commensurate rise in circus technology is the Neolithic Revolution, which is widely understood as an agricultural transition, moving humanity from hunting and gathering to farming. The Neolithic Revolution likely began in the fertile crescent around 10,000-8000 BC and in other places between 10,000 and 5,000 BC. It brought the domestication of plants and animals and led largely nomadic groups to create agrarian-based permanent settlements.

##### *4.1.1. Technology Change*

The Neolithic Revolution is generally characterized as a revolution in the production of food and the dissemination of new farming techniques. As Bocquet-Appel (2011) writes, “The major change that arose from this ‘revolution’ was, in evolutionary time, the number of potential mouths it was possible to feed per km<sup>2</sup>” (560). When distinguishing the Neolithic from other eras, scholars have used the concept of the “Neolithic Package”, which is the collection of elements common across space and particular to the Neolithic period (Çilingiroglu, 2005). There is some debate as to what, precisely, the “package” contains.<sup>17</sup> However, the list of contents is generally seen as including pottery, cultigens and domesticates. In contrast, there is little evidence that circus technology advanced as much as bread technology during the Neolithic Revolution.

##### *4.1.2. Model Implications*

According to our theory, a bread technology improvement where there is no improvement in circus technology should lead to growth in population density and a fall in living standards. That population density would increase after an improvement in bread technology is also accommodated by a standard single-good Malthusian model. In that model living standards would first rise then fall back to their original level - the “Malthusian trap”. In the single-good model, however, there is no possibility of a long-term decline in living standards. Living standards can fall to pre-shock

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<sup>16</sup>It is important to acknowledge that we are selecting data based on the magnitude of shocks.

<sup>17</sup>There is also some debate about the use of the terms since it may obscure variation across space in how the Neolithic Revolution played out.

subsistence and no further. Our two-good model differs in that it predicts that living standards can fall below pre-shock levels since there are fewer circuses to go around, but more mouths to feed.

#### 4.1.3. Evidence

A key prediction of our model is that a rise in bread production is followed by an increase in population density. It is well known that population levels and density rose during the Neolithic Revolution as individuals formed permanent settlements around farms (Bocquet-Appel, 2011). In terms of the magnitude, Shennan et al. (2013) discuss evidence of a six-fold rise in population density as a result of improvements in agricultural technology over the course of the Neolithic Revolution. Kremer (1993) argues that population could have risen much more.

A number of authors have claimed that living standards fell after the Neolithic Revolution. In light of post-Revolution falls in living standards, Diamond (1987) provocatively calls the adoption of agriculture “the worst mistake in the history of the human race”. We find this view somewhat shortsighted since it is difficult to argue that today’s living standards are worse than those of hunter-gatherers. However, in the aftermath of the Neolithic Revolution there is considerable evidence of falling living standards.<sup>18</sup> Guzmán and Weisdorf (2011) point to evidence of worse health and nutrition and longer working days among early adopters of agriculture in comparison to hunter-gatherers and Rowthorn and Seabright (2010) claim that scarce resources needed to be diverted to defending agricultural land. Shennan et al. (2013) use cemetery data to establish increased mortality, which they attribute to lack of clean drinking water and infectious disease as villages were formed and people were bound to farms and were therefore compelled to live in close proximity to one another. Related, Bandy and Fox (2010) claim that the advent of villages led to higher levels of social conflict. Higher mortality rates did not outweigh rising population. However, they do provide some evidence of lower living standards.<sup>19</sup>

It is important to point out that later technology shocks would alleviate some of the pressures associated with permanent settlements based around agriculture. Innovations in sanitation technology would prevent disease, developments in architecture would improve dwellings and social innovations would lead to better tools to resolve conflicts. Moreover, per capita income would eventually explode during the Industrial Revolution, which may never have occurred absent agriculture and permanent settlements. Therefore, later technology shocks cast doubt on the claim that agriculture was a “mistake”. Still, there is good evidence of falling living standards following a positive shock to food production. This pattern is difficult to reconcile with a standard single-good Malthusian model, which would instead predict that living standards would fall back to subsistence following population growth after a technology shock. In contrast, our model with two sectors accommodates the possibility that living standards (in our model: circuses per capita) could fall if bread technology supports a larger population left with fewer non-food goods per capita.

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<sup>18</sup>For an early example, Shand (1974) discusses the adoption of agriculture as an “agonizing transition”.

<sup>19</sup>Other researchers have asked why individuals would adopt the new technology in the first place given the costs it imposed on early adopters (Weisdorf, 2009).

## 4.2. *The Classical Period: Ancient Rome*

We now focus our attention on Ancient Rome, in particular, the first 100 years A.D. The reason is that there is good evidence of progress in technologies related to the circus sector, but also evidence that bread technology remained relatively unchanged.

### 4.2.1. *Technology Change*

Finley (1999) provides accounts of continuous progress in grape and olive-pressing equipment during this period in Rome. Moreover, Greene (2000) reports that fieldwork projects around the Mediterranean have shown that these technologies were applied throughout the region during Roman times. Here, it is important to note that grapes and wine are not goods that would fit into our bread sector since they are not necessary for survival and should not therefore be seen as governing population dynamics. Instead these are goods that improve the quality of life and are therefore in the circus sector.<sup>20</sup> Other evidence of non-agricultural innovation includes water power, use of pumps in mining, mass production of pottery, bricks, glass and paper in factories, aqueducts, fountains and baths.

At the same time, Greene (2000) states that “Finley...saw no selective breeding and no changes in tools or techniques for ploughing, harvesting or irrigation, although he did acknowledge modifications of land use”. In other words, there is evidence of large improvements in technologies related to goods in the circus sector and also evidence that technology in the bread sector remained about the same, relying on old techniques.

### 4.2.2. *Model Implications*

According to our theory, a circus technology improvement where there is little improvement in bread technology should lead to no growth in population density and a rise in living standards. Notice that a standard single-good model with a Malthusian constraint only permits short-run improvements to quality of life in the form of higher per capita income until population growth catches up, consuming output in excess of what is needed for subsistence. In contrast, our model accommodates the idea that a circus technology shock can improve living standards if there is no commensurate rise in bread technology. The reason is that the economy cannot sustain additional mouths to feed, but can produce more utility-enhancing goods per person.

### 4.2.3. *Evidence*

Some economic historians argued that the baths, monuments, colosseums, and palaces that Roman civilization was famous for were only available to a select few while ordinary individuals were just as poor as their counterparts in any other ancient society.<sup>21</sup> There are two issues with this argument. First, from a theoretical point of view, per capita income includes the income of the

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<sup>20</sup>Sweets would be another example. Hersh and Voth (2009) show that sugar, tea and coffee - luxuries by almost any measure, even if they are technically food - added the equivalent of over 16% of household income to British welfare by the eighteenth century.

<sup>21</sup>A similar point is made in Temin (2013).

very rich - the huge increase in world per capita income during the Industrial Revolution (discussed in more detail below) is a huge increase in mean income - median income changed very little until the 1980s. Second, more recent research has shown that the scale of Roman production of consumer goods was beyond what a tiny fraction of elites could use. For example, near the River Tiber, there is a mound of pottery waste, known as Monte Testaccio. The mound is composed of the fragments of an estimated 53 million *amphorae*, which once contained 1.6 billion gallons of olive oil that was imported to a nearby port in ancient times. No elite class could have consumed so much oil. Though anecdotal, this piece of evidence is in line with what scholars of Ancient Rome have carefully established: that the average Roman lived a far better life than the average European in the post-Roman period. As Ward-Perkins (2005) points out, for example, the poorest Roman rural houses had tiled roofs, something unavailable even to the elites of the post-Roman era. What's more, pottery and tiles are heavy and difficult to move around. If they can be produced in such a large scale, and traded across thousands of miles as archaeological findings suggest, it stands to reason that other consumer goods were traded, too, but are simply found less frequently because they are more perishable.

Further evidence on the Roman economy comes from the demand for copper currency. The mining of copper emitted pollution that was captured in ice in Greenland. In fact, ice data reveal the Roman period as one of the three peaks of world-wide copper production (Hong et al. (1996)). The contrast between the peaks and the time in between is striking. Immediately after Rome collapsed, world-wide copper production fell to less than a seventh of its previous level. A summary account of the Roman economy and the prosperity it brought to common Romans can be found in Temin (2013), who argues that Roman Italy was comparable to the Netherlands in 1600 and estimates per capita GDP over the entire Roman Empire at \$1000 1900 dollars.

If, in Roman times, there was an improvement in circus technology and no commensurate improvement to bread technology, we would expect population density to remain relatively flat. Indeed, when we look at evidence from Italy, we find little evidence of explosive population growth. In particular, Frier (2000), using data from a variety of sources, including McEvedy, Jones et al. (1978) shows that population in Italy in A.D. 14 was 7 million (with a land mass of 2,500,000 km<sup>2</sup>). In A.D. 164, population had risen to 7.6 million. Population density (per km<sup>2</sup>) is calculated at 28 in A.D. 14 and 30.4 in 164. That Rome developed technologically and population did not rise casts doubt on the assumption of a single composite good subject to a Malthusian constraint as a valid way to model historical growth. The lack of population growth in Italy during the Roman era is important for a second reason: Rome did eventually fall, so it is possible to argue that traditional Malthusian forces were at work and high per capita income was simply a transitory phenomenon until Malthusian forces caught up. The problem with this argument is that to be true it would have to be the case that population density was rising so that eventually population outstripped food supply. There are many reasons for the fall of Rome - but increased population density in Italy is

not one of them.<sup>22</sup>

### 4.3. *The Classical Period: Song China*

We have argued that Ancient Rome provides a counterexample to the view that sustained rises in living standards occurred first during the Industrial Revolution. Here, we argue that the Song dynasty in China offers another. The period we consider began in 960 and ended in 1127. The period saw a flowering of culture, science and commerce - along with a rise in population.

#### 4.3.1. *Technology Change*

China during the Song dynasty is generally characterized by a number of technological advancements, both in agriculture and in other sectors. For agriculture, there were increases in plowable land and improvements to irrigation. In other sectors, there were a number of developments, including gunpowder, the use of coal as fuel, improvements in iron and steel production, the introduction of banknotes, the development of joint stock trading companies and improved international commerce.

#### 4.3.2. *Model Implications*

In our model, a sufficiently large improvement in technology should lead to rises in population, but has an ambiguous impact on living standards. If the shock to  $B$  dominates, living standards would be expected to fall (as is the case with the Neolithic Revolution). If  $C$  dominates, living standards would rise.

#### 4.3.3. *Evidence*

There is evidence that population grew in Song China, roughly doubling from about 55 million to 100 million (Maddison, 2003). However, in Maddison's data (Maddison, 2003), which relies largely on food intake, Song China's GDP per capita is estimated to be \$450, and Ming China's is estimated at \$600. This modest rise may not tell the whole story. Like Ancient Rome, Song China enjoyed great prosperity in industry and commerce. Moreover, Song left reliable administrative records with which we can reconstruct economic life with some confidence. According to Liu (2015), only a third of the Song government's tax revenue came from agriculture. The other two thirds were from commerce and manufacturing. In comparison, agriculture contributed as much as 84% to the Ming government's tax receipts. The difference is not because Song levied lower taxes on agriculture. Just the opposite, Song's agricultural tax was even higher than Ming's. Its total tax was three times as large because Song's industry and commerce were highly developed. The relative importance of the Chinese dynasties and their abundant records tell us that Maddison's estimates may not be correct. This may explain why Marco Polo marveled at Chinese civilization (he arrived when the Mongols were about to conquer the Southern Song), whereas later European visitors to Ming and Qing China were hardly as taken by what they saw. This is not surprising. In the intervening

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<sup>22</sup>In relatively underpopulated areas, particularly France and Spain, the adoption of Roman agricultural technology did lead to a substantial increase in population density.

years, Europe developed and China declined. The rise and fall were not in the absolute size of the economy - population grew in both ends of Eurasia during the time in between - but in the ratio of industry to agriculture, which determined average living standards.

#### *4.4. Late Medieval Period: The Great Divergence*

The Great Divergence describes shifts in relative per capita income in China versus Europe (specifically England) during the late Medieval period. It is known as a “divergence” since per capita income soared in Europe and not in Asia (Allen, 2001) even though China and Europe looked fairly similar economically circa 1750 (see for example, Pomeranz (2009)). The period includes the Industrial Revolution, which we will discuss below. It also coincides with the introduction and adoption of maize in China between the 1500s and the 1900s, which we argue played an important role in the divergence.

Generally, the Great Divergence is explained as soaring circus technology in Europe contrasted with stagnant technology in China. Here, we offer a novel perspective based on data from Ireland. To understand, first note that our model accommodates the idea that a positive shock to technology can lead to a fall in living standards. This can occur in spite of a positive circus technology shock if this is accompanied by a large bread technology shock. We argue that this dynamic occurred not only in China, but in Ireland when the potato was introduced, which greatly increased the number of people who could be fed. Therefore, despite the likelihood that Ireland had access to the same circus technology as (nearby) England, Ireland grew poorer than England.

In fact, the divergence in Ireland looks exactly like that of China: there was a huge increase in total GDP similar to England, but with declining per capita income and greatly increased population. Apparent similarities between Ireland and China lead us to a reinterpretation of the Great Divergence between China and England. We discuss evidence showing that China imported New World crops, specifically maize, and that this led to higher population growth and lower living standards. This evidence provides a rather different perspective on the Great Divergence, which is usually interpreted as China falling behind; it suggests that the problem was that China pulled ahead - in agriculture.

##### *4.4.1. Technology Change*

Between 1500 and 1900, roughly corresponding to the Qing dynasty, both China and Europe incorporated New World crops. In the case of China, maize became a staple crop. However, there is little evidence of growth in other technologies. In fact, in his very detailed study of the Great Divergence, Pomeranz (2009) argues against what he calls the Weberian view - that China grew differently than Europe because of some cultural differences that favored growth in one place over the other. Rather, the argument is that the location of coal deposits and the discovery of the New World (exogenous shocks to production technology, especially manufacturing) meant that Europe was able to produce more goods, but China was not. Thus, the period in China is an example of a rise in bread productivity (via the incorporation of maize) without a commensurate rise in circus productivity.

Similarly, in Ireland, a more agrarian economy, the potato seemed to play an outsized and early role in the economy. Not only was it widely adopted in Ireland, but it also led to relatively large changes in food output. The potato more than doubled caloric output - Mokyr (1981) claims that it quadrupled output in Ireland. Moreover, Mokyr (1981) points out that introduction of the potato may have led to further improvements in bread technology, encouraging the development of practices such as crop rotation, which would further exacerbate the discrepancy between bread and circus technology.

In summary, there was a large positive shock to bread technology in China and also in Ireland. In England there is a large shock to circus technology but not bread technology. This circus technology was also available in Ireland, but perhaps not in China.

#### *4.4.2. Model Implications*

In Ireland and China, population should go up but per capita income should go down or up less quickly. The model predicts a greater increase in population, but lower living standards in Ireland and China compared to England.

#### *4.4.3. Evidence*

The adoption of maize in China led to growth in population density. Causal estimates are provided in Chen and Kung (2011), who find using IV estimates that a decade of maize planting resulted in a 5.6 percent annual increase in population density between the period 1500-1900. Chen and Kung (2011) follow Acemoglu, Johnson, and Robinson (2001) and Acemoglu, Johnson, and Robinson (2005) in using urbanization as a way to measure income. They find some evidence of declines in urbanization following maize adoption in China. Recall, an implication of the standard Malthusian model is that population growth following a technology shock would drive living standards back down to subsistence. However, if urbanization reflects income, it would appear that maize adoption in China caused living standards to *decline* relative to their pre-shock levels, which is a prediction of our model and which is not accommodated by the standard model.

Turning to Ireland, Nunn and Qian (2011) show that potato cultivation led to explosive population growth in parts of the Old World, leading some regions, such as Ireland, to a Malthusian-like poverty trap. After the introduction of the potato, there is evidence that in comparison to England, population grew rapidly, but that per capita consumption did not. Between the early 1600s (about when the potato was introduced, see, for example, Nunn and Qian (2011)) until about 1800, Irish population grew from roughly between 500,000 and 1,000,000 people (depending on the source) to about 6,000,000 at the start of the 1800s. It should be noted that most sources put the starting point at the lower number, which would suggest a 12-fold increase in population. In England over the same period population rose 2-3 times, from 4.2 million to 10.5 million (see, for example, McEvedy, Jones et al. (1978)) Over the same time period, English per capita income roughly doubled from 1082 to 2108 in year 2000 US dollars (Bolt and van Zanden, 2013).<sup>23</sup> Unfortunately, we do not have

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<sup>23</sup>See also The Maddison-Project, <http://www.ggdc.net/maddison/maddison-project/home.htm>, 2013 version.

estimates in per capita consumption in Ireland over the same period. The numbers are not found in the updates to the Maddison data set, including Bolt and van Zanden (2014). Never-the-less contemporary accounts indicate that Ireland (compared to England) remained mired in poverty and insofar as there was per capita growth it was not on the English scale, nor did it accrue to the Irish as most of Ireland's wealth (in the form of land) was controlled by and benefitted the English (Moody, Martin, and Byrne, 1976).

#### *4.5. The Industrial Revolution*

We now turn our attention to the Industrial Revolution. We argue that the Industrial Revolution was the result of technology shocks to both circuses and bread, where the former dominated. In single-good models it is necessary to introduce non-Malthusian demographic forces to explain why the Industrial Revolution raised per capita income. Our model is consistent with the same demographic forces being at work before and after the Industrial Revolution.

The Industrial Revolution lasted from about 1760 to 1820 and was the period when a largely agrarian economy was replaced by one where industry and manufacturing dominated. The Industrial Revolution is generally seen as the first period in history when living standards rose sustainably - though other work sees rises in living standards for the median individuals occurring much later. What is less controversial is the nature of changes that occurred in the economy. Whereas the shift in technology during the Neolithic Revolution mostly improved bread production, in the Industrial Revolution, it was the productivity of sectors for goods that were not essential to survival that rose.

##### *4.5.1. Technology Change*

The argument is not that bread technology did not improve; it did as agriculture and transportation improved. The argument is that circus technology improved more. Several papers report factor productivity for the economy as a whole and for agriculture in particular. For example, Feinstein (1981) reports that TFP growth in all sectors was roughly 1.3% per year after 1800. In contrast, Kögel and Prskawetz (2001) and Crafts (1980) report TFP growth in agriculture to be 0.9% per year between 1820-1840. Given the relatively low share of manufacturing in the economy, generating TFP growth of this magnitude for the entire economy would require large relative growth in TFP in manufacturing versus agriculture.

Rises in living standards do not generally (and should not be expected to) accompany rises in food consumption. The key is to divorce food consumption with the consumption over other goods. To illustrate this point, we turn once more to modern data. We consider growth in the dollar value of musical instruments per person, again from 1929 to 2000, in comparison to food (Figure 2, normalized to \$1.00 and 1 calorie per person, respectively, in 1929). Calorie growth is flat. However, the dollar value of the stock of musical instruments per person is anything but flat, a fact that likely did nothing to improve survival, but certainly led to an improvement in the quality of life for many people.



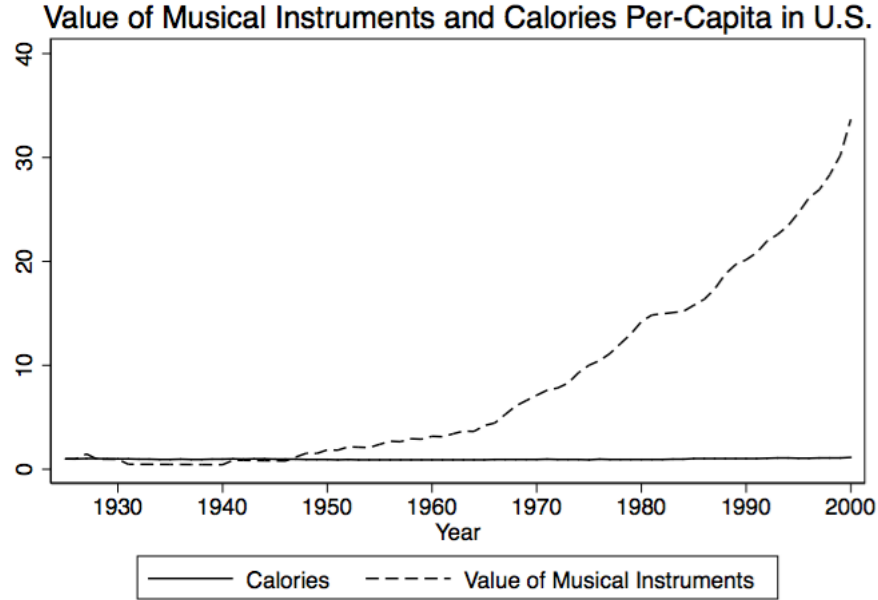


Figure 2: Calories per capita in the U.S. from 1909-2000.

#### 4.5.2. Model Implications

The simple two-sector model leads to a straightforward explanation of what took place: large and positive shocks to circus technology absent commensurate rises in bread technology lead to widespread and explosive growth in living standards.

#### 4.5.3. Evidence

The Industrial Revolution led to massive and sustained rises in living standards that were unprecedented in human history. Population rose as well, but not so much as to push humanity back to subsistence. Finally, there is little evidence that food consumption per capita changed very much (though Fouquet and Broadberry (2015) show evidence that the quality of calories may have improved).

Much research is dedicated to the project of making sense of the Industrial Revolution in light of centuries of slow and sporadic growth. We believe that our model offers a unified explanation that accommodates various episodes throughout history that other models have trouble explaining. Technology growth in either the bread or the circus sectors occurred throughout history. Many shocks were such that additional output would be consumed by larger populations. In other cases, living standards declined since circus technology did not improve as quickly as bread technology (for example, due to the introduction of new crops well suited to the climate). Finally, in other cases, living standards improved since circus technology improved more rapidly than bread technology. This occurred on a small but significant scale in Ancient Rome and on a massive scale during the Industrial Revolution.

## 5. Historical Evidence for Model Implications: Negative Shocks

To test model implications, we also study two negative shocks: the Black Death and the Irish Potato Famines.

### 5.1. *The Irish Potato Famine*

Here, we return to Ireland and potato cultivation, focussing on the Irish Potato Famine that occurred between 1845 and 1852. In the popular view of Malthus, population outstrips the food supply leading to famine which reduces the population. The Irish famine is sometimes used to illustrate this point. As should be clear, however, the potato famine was not caused by overpopulation, but by an exogenous shock - a crop disease (potato blight) that occurred world-wide (Donnelly Jr, 2012; Mokyr, 1981). The same observation can be made about the earlier famine in Ireland in 1740-41 and no doubt an event that influenced Malthus's writing in 1798. Like the better known potato famine, the earlier Irish famine has no causal link to population: it was caused by an exogenous change in climactic conditions - unusually cold winters - it was also a negative technological shock.

The idea is not that Ireland lacked access to circus technology from England, which is difficult to assume given proximity. Rather, Ireland was well-suited for potato cultivation which means that it experienced a larger relative shock to bread technology and, as our two sector model implies, it is the relative size of these shocks that matters. Therefore, by the time of the blight (the negative shock to bread technology), Ireland had a large population sustained by a single crop characterized by low living standards.

#### 5.1.1. *Technology Change*

In our model, the potato blight is a negative technology shock to  $B$ : it lowered the amount of food that could be produced with a given amount of labor and land. It was not due to overpopulation, but was likely due to reliance on a single strain, which was an inexpensive way to sustain a large population. In other words, a third factor (reliance on a single strain) made sustaining a large population possible and also likely led to the blight (Fraser, 2003).

#### 5.1.2. *Model Implications*

A drop in  $B$  with  $C$  fixed lowers  $\beta$  and also raises  $\zeta$ . This implies that in the long run population must fall but per capita income should rise. The model also implies that a high  $B$  economy with most land and labor dedicated to the agricultural sector is very vulnerable to  $B$  shocks since the shock cannot be offset by shifting land or labor from the circus sector. Hence population has to adjust very quickly.

#### 5.1.3. *Evidence*

The evidence supports both the short and long term effects of a drop in bread technology. The theory predicts that a consequence of the famine is not only that population should decline, but per capita GDP should go up. During the famine about 1 million people starved and another million emigrated so that Ireland lost about 25% of its population (Woodham-Smith and Davidson, 1991).

Maddison (2003) contains per capita GDP in Ireland and England before and after the famine. The per capital GDP in year 2000 US dollars for Ireland for 1820 and 1870 are 877 and 1775. Comparable numbers for England are 2074 and 3190, respectively. Together, population decline and rapid growth in per capita GDP in Ireland versus England seem to support the predictions of the model.<sup>24</sup> This point deserves some emphasis: our model predicts that a consequence of a negative technological shock to the bread sector not only lowers population but increases per capita income. This is not such an obvious conclusion - yet in Ireland a little noted consequence of the potato famine is an enormous increase in per capita income.

## 5.2. *The Black Death*

A canonical example often used to support the one-good Malthusian model is the Black Death plague which led to a massive decline in population. The reason is that there is evidence that the surviving population had higher living standards after the plague had subsided. The plague reached its peak in the mid 1300s and resulting deaths are estimated at 75-200 million (Gottfried, 2010).

### 5.2.1. *Technology Change*

The shift is not a technology change, but instead is a negative shock to the population coupled with no change to technology.

### 5.2.2. *Model Implications*

Our model implies that a decline in population should be followed by population growth. This is what the single-good model would imply. Given prevailing technology and the fact that land is fixed and that marginal returns decline, more individuals could be sustained. In other words, the standard model would predict more food production until the Malthusian constraint is met. Until then, people consume more, but living standards return to subsistence. In contrast, our model accommodates the idea that fewer individuals were needed to produce food to sustain the smaller population. These individuals shifted, therefore, to circus production, which raised living standards.

### 5.2.3. *Evidence*

There is ample evidence of population recovery following the Black Death, which is in line with our model predictions. Model implications also accommodate the flourishing of art, culture and science, i.e., rises in living standards that occurred after the Black Death (Herlihy and Cohn, 1997; Pamuk, 2007; Clark, 1998). Interestingly, Clark (1998) sees little evidence of greater agricultural productivity until the mid 1500s, which would further suggest movement of labor to the circus sector.

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<sup>24</sup>It is worth pointing out that dynamic models in which the population effect works through the choice of number of children are not going to get this right. The decline in population in Ireland occurred through two mechanisms: starvation and emigration. Likewise, for historical episodes characterized by improvements in technology, immigration alongside fertility is an important factor.

## 6. Conclusion

The traditional Malthusian point of departure is the following: living standards were roughly the same everywhere in the world until about 1760. As measured by per capita GDP, living standards, as recent research has shown and we have documented, have varied enormously over time and space. We do not argue with the well-established observation that growth after the Industrial Revolution has been rapid and sustained, dwarfing earlier growth, which was comparatively sporadic and slow. What we do argue against is the subsequent jump to approximating earlier growth as no growth, which we view as an over-simplification leading to theories that must explain the Industrial Revolution as arising from a shift in fundamentals rather than changes in technology. In other words, we argue that early growth should be taken at face value. And once we acknowledge fluctuations in living standards throughout history, we must also acknowledge that Malthusian theory does a poor job of explaining them - and even more so in the face of the Industrial Revolution.

We have proposed a simple two sector model according to which the Industrial Revolution is not seen as some sort of aberration in human behavior. Rather our theory says that improvements in bread technology will largely result in population growth while improvements in circus technology will largely result in growth in per capita GDP. We hope that this theory will lead to more serious work exploring ancient economic growth such as the recent work of Temin (2013). Indeed - as Temin (2013) says - the distinguishing feature of modern economic growth is not that it takes place, but rather that it takes place so quickly. In particular our model suggests a key role is played by  $\zeta$  as a measure of relative technological change in the bread and circus sector and it would be useful if economic historians made a systematic effort to measure this.

Perhaps it is useful to conclude by indicating what we hoped to accomplish by writing this paper and what we learned in the course of writing it. We started with the idea that in a two sector model it makes a difference whether technological shocks hit the bread or circus sector, with the former favoring population growth and the latter growth in per capita GDP. We constructed a simple two sector Malthusian model and verified that it indeed has this property. This accomplished our goal of accommodating the basic observation that prior to the Industrial Revolution big changes in technology - generally favoring agriculture - had little impact on living standards while the opposite was the case in the Industrial Revolution. In those historical cases where technology did favor circuses we find that indeed per capita income did increase. Other data support the basic validity of the model: for example the fact, perhaps not so well known, that the population density of Italy changed little during the Roman period. Without the model we would not have thought to consider the impact of the Irish Potato Famine on per capita GDP; we simply assumed that it had a bad effect. In fact, as the model predicts, it went up quite a lot.

We certainly did not set out to explain the Great Divergence - rather were puzzled by the implication of the model that a big increase in circus technology, if offset by an even larger increase in bread technology, would not lead to increased per capita income, but would rather be reflected in increased population. In discussions of the Great Divergence, the great increase in Chinese population is noted, along with the fact that this was fueled in large part by improvements in

agricultural technology brought about by the import of New World crops. By contrast most of Europe showed little impact from New World crops due to unsuitable growing conditions. This dovetails with the theory. Moreover: when we went to look at places in Europe where New World crops did have a big impact - Ireland - we discovered the same story as China. In Ireland, population grew much faster than in England and per capita GDP rose much more slowly. Hence, in light of our model, we argue that the Great Divergence is less due to the unavailability of new circus technology in China, but more to the perverse effect of improved bread technology.

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## Appendix A. Proofs for Propositions

*Summary of the technology*

The utility function is

$$U(x, y) = \frac{x^{1-\rho} - 1}{1-\rho} + \frac{y^{1-\sigma} - 1}{1-\sigma}$$

the production function

$$x = BL_B^{1-\gamma_B} N_B^{\gamma_B} / Ny = CL_C^{1-\gamma_C} N_C^{\gamma_C} / N$$

and the resource constraints  $L_B + L_C = L$ ,  $N_B + N_C = N$ .

*First Order Conditions for Land/Labor Allocation*

Substitute the production technology into the utility function and write  $N_C = N - N_B$ ,  $L_C = L - L_B$  and differentiate with respect to  $N_B$  and  $L_B$  to get two FOC equations for the utility maximizing allocation of land and people between sectors are

$$(1-\gamma_B) \left( \frac{BL_B^{1-\gamma_B} N_B^{\gamma_B}}{N} \right)^{-\rho} B \left( \frac{N_B}{L_B} \right)^{\gamma_B} \frac{1}{N} = (1-\gamma_C) \left( \frac{C(L-L_B)^{1-\gamma_C} (N-N_B)^{\gamma_C}}{N} \right)^{-\sigma} C \left( \frac{N-N_B}{L-L_B} \right)^{\gamma_C} \frac{1}{N}$$

and

$$\gamma_B \left( \frac{BL_B^{1-\gamma_B} N_B^{\gamma_B}}{N} \right)^{-\rho} B \left( \frac{L_B}{N_B} \right)^{1-\gamma_B} \frac{1}{N} = \gamma_C \left( \frac{C(L-L_B)^{1-\gamma_C} (N-N_B)^{\gamma_C}}{N} \right)^{-\sigma} C \left( \frac{L-L_B}{N-N_B} \right)^{1-\gamma_C} \frac{1}{N}$$

These can be rewritten as

$$x^{1-\rho} \frac{1-\gamma_B}{L_B} = y^{1-\sigma} \frac{1-\gamma_C}{L_C}$$

$$x^{1-\rho} \frac{\gamma_B}{N_B} = y^{1-\sigma} \frac{\gamma_C}{N_C}$$

*Summary of Equations*

Two first order conditions for the utility maximizing allocation of land and people between sectors

$$y^{1-\sigma} = \frac{1-\gamma_B}{1-\gamma_C} x^{1-\rho} \frac{L_C}{L_B}$$

$$y^{1-\sigma} = \frac{\gamma_B}{\gamma_C} x^{1-\rho} \frac{N_C}{N_B}$$

the production technology

$$x = BL_B^{1-\gamma_B} N_B^{\gamma_B} / N$$

$$y = CL_C^{1-\gamma_C} N_C^{\gamma_C} / N$$

the feasibility constraints  $L_B + L_C = L$ ,  $N_B + N_C = N$  and the Malthus equilibrium condition  $x = \bar{x}$  or the short term constraint  $N$ . The variables are:  $L_B, L_C, N_B, N_C, y, N, x$  with 7 equations.

Equate the two FOCs to find the *ratio equation*

$$N_C/L_C = \left( \frac{(1-\gamma_B)\gamma_C}{(1-\gamma_C)\gamma_B} \right) N_B/L_C = \nu N_B/L_B$$

where notice that since  $\gamma_C > \gamma_B$  we must have  $\nu > 1$ . This may also be written as

$$N_C/N_B = \nu L_C/L_B$$

which shows that both land and labor shift sectors in the same direction and that the circus to bread labor ratio is more responsive than the land ratio.

Take the root of the first FOC to get the *basic root FOC*

$$y = x^{\frac{1-\rho}{1-\sigma}} \left( \frac{L_C}{L_B} \right)^{\frac{1}{1-\sigma}} \left( \frac{1-\gamma_B}{1-\gamma_C} \right)^{\frac{1}{1-\sigma}}$$

Solve the first production equation for  $N$  and plug into the second production equation to get the *first general production equation*

$$y = x \frac{CL_C^{1-\gamma_C} N_C^{\gamma_C}}{BL_B^{1-\gamma_B} N_B^{\gamma_B}} = x \left( \frac{C}{B} \right) \left( \frac{L_C}{L_B} \right) \frac{(N_C/L_C)^{\gamma_C}}{(N_B/L_B)^{\gamma_B}}$$

Use the ratio equation to find the *second general production equation*

$$y = x \left( \frac{C}{B} \right) \left( \frac{L_C}{L_B} \right) \nu^{\gamma_C} \left( \frac{N_B}{L_B} \right)^{\gamma_C - \gamma_B}$$

*Main Proposition for the Malthus Case*

There is a unique Malthusian equilibrium with the following comparative static properties

1.  $y$  is increasing in  $\zeta$
2. Population is strictly increasing without bound in  $\beta$ . So, given an initial  $\beta$  and  $\zeta$  and a technology shock  $\zeta'$ , there exists  $\bar{\beta}$  such that  $N' > N$  iff  $\beta' > \bar{\beta}$ . Moreover,  $\bar{\beta} \leq \beta$  iff  $\zeta' \leq \zeta$ .

*Solving the Malthus Equilibrium*

Replace  $x$  with  $\bar{x}$  everywhere to get rid of one equation and one variable, take the population constraint  $N_B + N_C = N$  and write it as

$$\frac{N_B}{L_B} \frac{L_B}{L_C} + \frac{N_C}{L_C} = \frac{N}{L_C}$$

and use the second production equation to get rid of  $N$

$$\frac{N_B}{L_B} \frac{L_B}{L_C} + \frac{N_C}{L_C} = \frac{CL_C^{1-\gamma_C} N_C^{\gamma_C}}{yL_C} = \frac{C(N_C/L_C)^{\gamma_C}}{y}$$

rewritten as

$$y = \frac{C(N_C/L_C)^{\gamma_C}}{\frac{N_B L_B}{L_C} + \frac{N_C}{L_C}}$$

and for  $N_C/L_C$  use the ratio equation to get it in terms of  $N_B/L_B$  to get the *third equation*

$$y = \frac{C\nu^{\gamma_C}(N_B/L_B)^{\gamma_C-1}}{\frac{L_B}{L_C} + \nu}$$

Summarizing we now have the root of the first FOC, the second general production equation and the third equation

$$y = \bar{x}^{\frac{1-\rho}{1-\sigma}} \left(\frac{L_C}{L_B}\right)^{\frac{1}{1-\sigma}} \left(\frac{1-\gamma_B}{1-\gamma_C}\right)^{\frac{1}{1-\sigma}}$$

$$y = \bar{x} \left(\frac{C}{B}\right) \left(\frac{L_C}{L_B}\right) \nu^{\gamma_C} (N_B/L_B)^{\gamma_C-\gamma_B}$$

$$y = \frac{C\nu^{\gamma_C}(N_B/L_B)^{\gamma_C-1}}{\frac{L_B}{L_C} + \nu}$$

and three variables to solve for  $y, N_B/L_B, L_C/L_B$ .

Next we get it down to two. We take the second general production equation and the third equation and use them to eliminate  $y$  to get

$$\bar{x} \left(\frac{C}{B}\right) \left(\frac{L_C}{L_B}\right) \nu^{\gamma_C} (N_B/L_B)^{\gamma_C-\gamma_B} = \frac{C\nu^{\gamma_C}(N_B/L_B)^{\gamma_C-1}}{\frac{L_B}{L_C} + \nu}$$

and solve this for  $N_B/L_B$  to find

$$N_B/L_B = \left[ \frac{C\nu^{\gamma_C}}{\left(\frac{L_B}{L_C} + \nu\right) \bar{x} \left(\frac{C}{B}\right) \left(\frac{L_C}{L_B}\right) \nu^{\gamma_C}} \right]^{\frac{1}{1-\gamma_B}} = \left[ \frac{\nu^{\gamma_C}}{\left(1 + \nu \frac{L_C}{L_B}\right) \bar{x} \left(\frac{1}{B}\right) \nu^{\gamma_C}} \right]^{\frac{1}{1-\gamma_B}}$$

plug this back into second general production equation to eliminate  $N_B/L_B$  to get the *upper equation*

$$y = \bar{x} \left(\frac{C}{B}\right) \left(\frac{L_C}{L_B}\right) \nu^{\gamma_C} \left[ \frac{C\nu^{\gamma_C}}{\left(1 + \nu \frac{L_C}{L_B}\right) \bar{x} \left(\frac{C}{B}\right) \nu^{\gamma_C}} \right]^{\frac{\gamma_C-\gamma_B}{1-\gamma_B}} = \bar{x} \left(\frac{C}{B^{1-\frac{\gamma_C-\gamma_B}{1-\gamma_B}}}\right) \left(\frac{L_C}{L_B}\right) \nu^{\gamma_C} \left[ \frac{\nu^{\gamma_C}}{\left(1 + \nu \frac{L_C}{L_B}\right) \bar{x} \nu^{\gamma_C}} \right]^{\frac{\gamma_C-\gamma_B}{1-\gamma_B}}$$

together with the root of the first FOC

$$y = \bar{x}^{\frac{1-\rho}{1-\sigma}} \left(\frac{L_C}{L_B}\right)^{\frac{1}{1-\sigma}} \left(\frac{1-\gamma_B}{1-\gamma_C}\right)^{\frac{1}{1-\sigma}}$$

this gives two equations in two unknowns  $y, L_C/L_B$ .

The latter curve does not depend on the technology and is a power curve in  $L_C/L_B$  where the

power is bigger than 1, so in particular the slope at zero is zero and the function is convex.

#### *Concavity of the Upper Equation and Uniqueness*

The upper equation has the form  $\lambda/(1+a\lambda)^\alpha$  where  $0 < \alpha < 1$ . Differentiating we see

$$\frac{(1+a\lambda)^\alpha - \lambda\alpha a(1+a\lambda)^{\alpha-1}}{(1+a\lambda)^{2\alpha}} = \frac{1 - \lambda\alpha a(1+a\lambda)^{-1}}{(1+a\lambda)^\alpha}$$

and taking the second derivative we find

$$\alpha a \frac{-(1-\alpha)\lambda a - 2}{(1+a\lambda)^{\alpha+2}}$$

which is negative so that this function is concave.

#### *Analysis of the Malthus Case*

The root FOC is convex, the upper equation is concave hence there is a unique non-zero intersection of the two curves.

Only the upper equation depends on technology, and it depends only on

$$\frac{C}{B^{1-\frac{\gamma_C-\gamma_B}{1-\gamma_B}}} = \frac{C}{B^{\frac{1-\gamma_C}{1-\gamma_B}}} = \zeta^{1-\gamma_C}$$

and is increasing in this ratio. Hence increasing this ratio shifts the upper curve up increases  $y$  and conversely. Hence the same is true with respect to  $\zeta$ .

#### *Analysis of Population in the Malthus Case*

We look at the production function for  $x$

$$\bar{x} = BL_B^{1-\gamma_B} N_B^{\gamma_B} / N$$

$$\bar{x} = BL^{1-\gamma_B} \left( \frac{1}{1+L_C/L_B} \right)^{1-\gamma_B} \left( \frac{1}{1+N_C/N_B} \right)^{\gamma_B} / N^{1-\gamma_B}$$

Now use  $N_C/L_C = \nu N_B/L_B$

$$\bar{x} = BL^{1-\gamma_B} \left( \frac{1}{1+L_C/L_B} \right)^{1-\gamma_B} \left( \frac{1}{1+\nu L_C/L_B} \right)^{\gamma_B} / N^{1-\gamma_B}$$

$$N = \beta \left( L^{1-\gamma_B} \left( \frac{1}{1+L_C/L_B} \right)^{1-\gamma_B} \left( \frac{1}{1+\nu L_C/L_B} \right)^{\gamma_B} / \bar{x} \right)^{\frac{1}{1-\gamma_B}}$$

and note from above that  $L_C/L_B$  depends on the technology only through  $\zeta$  and is strictly increasing in  $\zeta$  so we may write  $N = \beta G(\zeta)$  where  $G(\zeta)$  is strictly decreasing. It follows directly that population is strictly increasing without bound in  $\beta$  and that  $\bar{\beta} \leq \beta$  iff  $\zeta' \leq \zeta$ , that is smaller  $\zeta$  increases population so lowers the threshold change in  $\beta$  needed to increase population.

*Solving the Short Run Equilibrium*

Rewrite the resource constraints

$$L_B + L_B \frac{L_C}{L_B} = L$$

$$L_B = \frac{L}{1 + L_C/L_B}$$

By symmetry

$$L_C = \frac{L}{1 + (L_C/L_B)^{-1}}$$

Using the ratio equation

$$N_B = \frac{N}{1 + N_C/N_B} = \frac{N}{1 + \nu L_C/L_B}$$

$$N_C = \frac{L}{1 + (N_C/N_B)^{-1}} = \frac{L}{1 + (\nu L_C/L_B)^{-1}}$$

We get the *second general production equation* using the above

$$y = x \left( \frac{C}{B} \right) \left( \frac{L_C}{L_B} \right) \nu^{\gamma_C} \left( \frac{N_B}{L_B} \right)^{\gamma_C - \gamma_B} = x \left( \frac{C}{B} \right) \left( \frac{L_C}{L_B} \right) \nu^{\gamma_C} \left( \frac{N(1 + L_C/L_B)}{L(1 + \nu L_C/L_B)} \right)^{\gamma_C - \gamma_B}$$

The root FOC is

$$y = x^{\frac{1-\rho}{1-\sigma}} \left( \frac{L_C}{L_B} \right)^{\frac{1}{1-\sigma}} \left( \frac{1 - \gamma_B}{1 - \gamma_C} \right)^{\frac{1}{1-\sigma}}$$

We combine these two

$$x \left( \frac{C}{B} \right) \left( \frac{L_C}{L_B} \right) \nu^{\gamma_C} \left( \frac{N(1 + L_C/L_B)}{L(1 + \nu L_C/L_B)} \right)^{\gamma_C - \gamma_B} = x^{\frac{1-\rho}{1-\sigma}} \left( \frac{L_C}{L_B} \right)^{\frac{1}{1-\sigma}} \left( \frac{1 - \gamma_B}{1 - \gamma_C} \right)^{\frac{1}{1-\sigma}}$$

$$x = \left[ \left( \frac{B}{C} \right) \left( \frac{L_C}{L_B} \right)^{\frac{\sigma}{1-\sigma}} \left( \frac{L(1 + \nu L_C/L_B)}{1 + L_C/L_B} \right)^{\gamma_C - \gamma_B} \nu^{-\gamma_C} \left( \frac{1 - \gamma_B}{1 - \gamma_C} \right)^{\frac{1}{1-\sigma}} / N^{\gamma_C - \gamma_B} \right]^{\frac{1-\sigma}{1-\sigma+\rho}}$$

*Analysis of Monotonicity*

The function

$$\frac{1 + \nu x}{1 + x}$$

has first derivative

$$\frac{\nu(1 + x) - (1 + \nu x)}{(1 + x)^2} = \frac{\nu - 1}{(1 + x)^2} > 0$$

since  $\nu > 1$ . Hence this function is increasing since  $\rho > -(1 - \sigma)$ .

*Short-run Comparative Statics*

The production function for  $x$  using the results above is

$$x = BL_B^{1-\gamma_B} N_B^{\gamma_B} / N$$

which may be written as

$$x = B \left( \frac{L}{1 + L_C/L_B} \right)^{1-\gamma_B} \left( \frac{N}{1 + \nu L_C/L_B} \right)^{\gamma_B} / N = B \left( \frac{L}{1 + L_C/L_B} \right)^{1-\gamma_B} \left( \frac{1}{1 + \nu L_C/L_B} \right)^{\gamma_B} / N^{1-\gamma_B}.$$

This is decreasing in  $L_C/L_B$ . The other equation from above is

$$x = \left[ \left( \frac{B}{C} \right) \left( \frac{L_C}{L_B} \right)^{\frac{\sigma}{1-\sigma}} \left( \frac{L(1 + \nu L_C/L_B)}{1 + L_C/L_B} \right)^{\gamma_C - \gamma_B} \nu^{-\gamma_C} \left( \frac{1 - \gamma_B}{1 - \gamma_C} \right)^{\frac{1}{1-\sigma}} / N^{\gamma_C - \gamma_B} \right]^{\frac{1-\sigma}{1-\sigma+\rho}}$$

which we know is increasing in  $L_C/L_B$ . This immediately implies a unique equilibrium.

An increase in  $B$  or decrease in  $N$  causes both functions shift up so bigger  $x$ .

If  $\gamma_C = \gamma_B$  and  $N$  decreases then only the decreasing function shifts up so  $L_C/L_B$  goes up

An increase in  $C$  makes  $x$  goes down and  $L_C/L_B$  goes up.

Note that with fixed  $L, N$  increased/decreased  $x$  must result in decreased/increased  $y$

The wage rate in bread units is the marginal of

$$Nx = BL_B^{1-\gamma_B} N^{\gamma_B}$$

with respect to  $N_B$ , that is

$$\gamma_B B (L_B/N_B)^{1-\gamma_B} = \gamma_B B \left( \frac{L(1 + \nu L_C/L_B)}{N(1 + L_C/L_B)} \right)^{1-\gamma_B}$$

so if  $\nu = 1$  (that is,  $\gamma_C = \gamma_B$ ) this is decreasing in  $N$ .

## Appendix B. Previous Research (Web Appendix)

In this web appendix, we summarize a number of papers on historical economic growth. We emphasize how most of these papers take flat historical economic growth for granted and then explain the Industrial Revolution as resulting from a shift in economic primitives. In nearly all cases, there is a single composite good subject to a Malthusian constraint. The main exception is preferences of offspring. Therefore, it is generally not the case that agents have preferences over goods that are not in some way tied up with survival, evolutionary fitness or population growth. There have been some exceptions to this general pattern, which are therefore closer to our contribution. We discuss those papers in the main text.

Table A.1: Economic Literature on Historical Growth

Paper	Technology	Preferences	Evidence	Findings
Acemoglu and Zilibotti (1997)	$Y_t = AK_t^\alpha L_t^{1-\alpha}$	$E_t U_t = \log(c_t)$ $\beta \int_0^1 \log(c_{t+1}^j) dj$ where $E_t$ is the expectations operator and $j$ is the state.	Braudel and Kochan (1973); Braudel (1982); North and Thomas (1973); De Vries (1976)	Early stages of growth are characterized by volatility, not just stagnation.
Arifovic, Bullard, and Duffy (1997)	$f(k(t)) = k(t)^\alpha$ where $k$ is the capital-to-effective-labor ratio and $t$ in parentheses denotes real time.	$U_t = \ln c_t(t) + \ln c_t(t+1)$ where subscript $t$ denotes generation and $t$ in parentheses denotes real time.	Maddison (1982) (table 1.2); Summers and Heston (1991)	Transition from stagnation to growth is a long, endogenous process.
Ashraf and Galor (2011)	$Y_t = AK_t^\alpha L_t^{1-\alpha}$	$U_t = (c_t)^{1-\gamma} (n_t)^\gamma$	McEvedy, Jones et al. (1978); Ramankutty et al. (2002); Michalopoulos (2012); Putterman (2008); Peregrine (2003)	Empirical analysis: until 1500 CE, superior productivity increased population and left living standards unchanged.

Table A.1 - continued on next page

Table A.1 - continued from previous page

Paper	Technology	Preferences	Evidence	Findings
Becker and Barro (1998)	NA	$U_0 = v(c_0, n_0) + a\psi(U_1, n_0)$ , where $U_0$ is parents' utility, $v$ is period utility and $U_1$ is child's utility.	NA	Implications of dynastic utility and parent altruism: long-term interest rates, child survival rates and altruism increased fertility and altruism; rate of technological progress decreases it.
Galor and Weil (2000)	$Y_t = L_t^\alpha (A_t T)^{1-\alpha}$	$U_t = c_t^{1-\gamma} (w_{t+1}, n_t, h_{t+1})^\gamma$ where $h$ is human capital per child.	Maddison (1982); Lee (1980); Chao (1986)	A unified model to capture population output and technology change from Malthusian to modern growth.
Goodfriend and McDermott (1995)	$Y = (e_Y h N)^{1-\alpha} \times \int_0^M [x(i)]^\alpha di$ where $e_Y$ is the fraction of time devoted to prod. of final goods, $M$ is the measure of intermediate goods and $x(i)$ is the quantity of good $i$ .	$\int_0^\infty n(t) \ln c(t) e^{-\rho t} dt$ where $n$ , here, is family size, $t$ in parentheses is calendar time and $\rho$ governs discounting of future consumption.	Maddison (1982)	Population growth leads to increases in specialization, which eventually activates a learning technology that initiates industrial growth.
Hansen and Prescott (2002)	$Y_{1t} = A_{1t} K_{1t}^\alpha N_{1t}^\eta \times T_{1t}^{1-\alpha-\eta}$ and $Y_{2t} = A_{2t} K_{2t}^\theta N_{2t}^{1-\theta}$ where subscripts denote production sectors for the same consumption good.	$U = \log c_{1t} + \beta \log c_{2,t+1}, \dots$ for consumption of goods 1 and 2.	Clark (1998); Wrigley (1997); Maddison (1991)	Stagnant to growing living standards occurs as profit-maximizing firms begin employing a less land-intensive production process.

Table A.1 - continued on next page



Table A.1 - continued from previous page

Paper	Technology	Preferences	Evidence	Findings
Jones (2001)	$Y_t = A_t^\alpha L_{Y_t}^\eta T_t^{1-\eta} \epsilon_t$ where $\epsilon_t$ is a shock.	$u = (1 - \mu) \frac{\tilde{c}_t^{1-\gamma}}{\gamma} + \mu \frac{\tilde{n}_t^{1-\psi}}{1-\psi}$ . where the tilde indicates deviations from the mean.	Maddison (1982); Lee (1980); Jevons (1896); Schoenhof (1903)	The single most important factor in the transition to modern growth is increased compensation (as a fraction of output) to inventors.
Kremer (1993)	$Y = Ap^\alpha T^{1-\alpha}$ where $p$ is population.	Single good; preferences left unspecified.	McEvedy, Jones et al. (1978); Deevey (1960); United Nations (various years)	Population spurs technology, which (according to Malthus) promotes population growth. Hence, high populations lead to population growth.
Lee (1980)	$A = \mu_0(\mu_1(T^{-\alpha} + (1 - \mu_1)N_A^{-\alpha}))^{-1/\alpha}$	Single good; preferences left unspecified.	Russel (1958); Phelps-Brown and Hopkins (1955); Kerridge (1953)	Short-run negative effects of population growth. Population swings explained by exogenous mortality rates, not as a response to technological change.
Lee (1988)	$Y = N^{1-\alpha} R^\alpha$ where $R$ denotes resources.	Single good; preferences left unspecified.	Russel (1958); Phelps-Brown and Hopkins (1955); Kerridge (1953)	Population-induced technological change and Malthusian growth generate accelerating growth.
Lucas (2002)	$f(x, l, k) = Ax^\alpha l^{1-\alpha-\eta} k^\eta$ , where $x$ and $k$ are land and capital per capita.	$u_t = f(c_t, n_t, u_{t+1})$ where subscript $t$ denotes a generation.	Johnson (1997); Kremer (1993); Maddison (various); McEvedy, Jones, et al (1978); Parente and Prescott (1993); Pritchett (1997)	Endogenous capital accumulation cannot alone explain the Industrial Revolution. Fertility and changes in returns to human capital are also important.

Table A.1 - continued on next page

Table A.1 - continued from previous page

Paper	Technology	Preferences	Evidence	Findings
Razin and Ben-Zion (1975)	$K_{t+1} = F(K_t - C_t, L_t)$ where $F$ is linear and homogenous; $F = r(K - C)^\alpha L^{1-\alpha}$	$V = U_t(c_t, \lambda_t U_{t+1}(c_{t+1}, \lambda_{t+1}, U_{t+2}(\dots)))$ , $t = 0, 1, \dots$ , where $\lambda$ is per capita births.	NA	Increases in capital productivity can lower the rate of population growth if current generations care about future generations' well being.
Voigtlander and Voth (2013b)	$Y_1 = A_1 L_1^\alpha T^{1-\alpha}$ $Y_2 = A_2 L_2$ where subscripts are sectors: 1 is food; 2 is other goods.	$u = (c_1 - \underline{c})^\gamma c_2^{1-\gamma}$ if $c_1 > \underline{c}$ $u = \phi(c_1 - \underline{c})$ if $c_1 \leq \underline{c}$	Clark (various); Maddison (various)	Population shocks like the Black Death can lead to high-income steady states.
Yang and Zhu (2013)	$Y_{1t}^{\text{OLD}} = T_t^{1-\alpha} (A_{2t} L_{2t})^\alpha$ $Y_{1t}^{\text{NEW}} = [T_t^{1-\alpha} (A_{2t} L_{2t})^\alpha]^{1-\eta} \times X_t^\eta$ $Y_{2t} = A_{2t} L_{2t}$ where subscripts are sectors: 1 is food; 2 is other goods. Food is produced with either OLD or NEW technology, where NEW takes an "industry supplied" good $X_t$ .	Food consumption is constant. Remaining income is spent on non-food items.	Clark (various); Maddison (2003); Wrigley (various)	Agricultural modernization ignites the transition to modern growth.

Equations are from the corresponding paper, where some notation has been altered to facilitate comparison across studies. The following notation holds in all equations (unless otherwise indicated):  $Y$ : output;  $K$ : capital,  $L$ : labor;  $T$ : land,  $A$ : technology (where subscripts indicate factor-specific technology);  $U$ : utility;  $c$ : consumption;  $n$ : number of children;  $w$ : wage;  $h$ : human capital per worker; and  $N$ : number of workers in the economy.  $t$ -subscripts indicate time. Where subscripts represent something other than time (e.g., generations, states or sectors) they are identified as such. Unless otherwise indicated, lower-case letters are per capita units. Finally,  $\alpha$ ,  $\eta$  and  $\theta$  are production parameters between 0 and 1.  $\gamma$  and  $\psi$  are consumption parameters between 0 and 1.