Appendix

Appendix 1 (Equivalence of the market clearing rules): Let us denote the number of shareholders (sellers) and non-shareholders (buyers) who have submitted a bid higher than the market clearing price p^* by n_s and n_b. It follows then that

	# of shareholders	# of non-shareholders
higher bids than p^*	n_s	n_b
lower bids than p^*	$M-n_s$	$n-M-n_b$
total	M	$n-M$

When the clearinghouse follows the first approach, *i.e.*, $p^* = \sup\ \{p : D(p) = S(p)\}$, market excess demand at p^* is equal to zero. That is, $n_b - (M - n_s) = 0$. When it follows the alternative approach, *i.e.*, arranging all bids in a descending order, the market clearing price is equal to the M^{th} highest bid. Since there should be M bids higher than p^*, $n_s + n_b = M$. Both approaches result in the same equation for market clearing. *Q.E.D.*

Appendix 2 (Equivalence of the buyer's and seller's strategy under price taking):
The optimal bid for buyer i in round τ given his private signal and market information Y^* is a solution to the following:

$$\underset{b}{Max}\, E[(V-p)\cdot 1_{\{b\geq p\}} \mid X_i = x, Y^*] \tag{A.1}$$

The maximization problem for seller i is

$$\underset{b}{Max}\, E[V \cdot 1_{\{b \geq p\}} + p \cdot 1_{\{b < p\}} \mid X_i = x, Y^*] \qquad \text{(A.2)}$$

Since $V \cdot 1_{\{p \leq b\}} + p \cdot 1_{\{p > b\}} = V \cdot 1_{\{b \geq p\}} + p \cdot (1 - 1_{\{b \geq p\}}) = (V - p) \cdot 1_{\{b \geq p\}} + p$, it becomes

$$\underset{b}{Max}\; E[(V - p) \cdot 1_{\{b \geq p\}} \mid X_i = x, Y^*] + E[p \mid X_i = x, Y^*] \qquad \text{(A.3)}$$

Since $E[p \mid X_i = x, Y^*]$ is not affected by their bid, a maximization problem (A.3) for seller i is equivalent to that for buyer i given in (A.1). Hence, the optimal bid for trader i is the same irrespective of the identity as a buyer or a seller. *Q.E.D.*

Appendix 3 (Updated bids do not change the equilibrium price): Suppose that each trader, observing the market clearing price p^*, submits an updated bid in the next round. The problem faced by a trader i who has signal x and price information p^* is to find a bid to solve the following:

$$\underset{b}{Max}\; E[U(V, X_i, b) \mid X_i = x, p^*]$$

Since they can infer Y from price information, their updated bid is equal to $\varphi(x, y)$. Traders who submit a bid higher than the market clearing price are those whose signal x is greater than y. Since the updated bid $\varphi(x, y)$ is still higher than the market clearing price $\varphi(y, y)$, this does not change a price determined in the first round. The same argument applies for traders whose signals are smaller than y. That is,

$$\varphi(x,x) < \varphi(x,y) < p^* \qquad \text{for} \quad x < y$$
$$p^* < \varphi(x,y) < \varphi(x,x) \qquad \text{for} \quad x > y$$
$$\varphi(x,y) = \varphi(x,x) = p^* \qquad \text{for} \quad x = y$$

A trader who tendered a bid higher (lower) than p^* will find it optimal to submit a new bid which is smaller (higher) than his initial bid but still higher (lower) than the market clearing price p^*. Price information does not affect a trader whose initial bid is p^*. Hence the updated bids do not change the market clearing price determined in the first round.

Appendix 4 ($\phi(Y) = \varphi(Y,Y)$ is a unique function satisfying (3.8)):
When $\phi(Y) = \varphi(Y,Y)$, $E[V \mid X_i = Y, p = \phi(Y)]$ is equal to $\phi(Y)$ since $E[V \mid X_i = Y, p = \phi(Y)] = \varphi(Y, \phi^{-1}(p)) = \varphi(Y,Y)$. Since $E[V \mid X_i = x, p = \phi(Y)]$ is increasing in x, $E[V \mid X_i = x, p = \phi(Y)] > (<)\ \phi(Y)$ for $x > (<)\ Y$.

Next, let us prove that the function satisfying (3.8) is unique. Suppose that it is not and there is another function $q(Y)$. Then, there should exist at least one point of $Y=y'$ such that $q(y') \neq \varphi(y',y')$. Since $q(Y)$ satisfies (3.8), $E[V \mid X_i = y', p = q(y')]$ should be equal to $\phi(y')$. It contradicts $q(y') \neq \varphi(y',y')$ since $E[V \mid X_i = y', p = q(y')] = \varphi(y',y')$. *Q.E.D.*

Appendix 5 (Naive traders' bidding strategy after circuit breakers have been triggered): By the same reasoning used in a proof of Theorem 2, the optimal bidding price of naive trader i as a solution to (3.15) is given as $\bar{b}_N = \varphi(x',x')$. As far as $\varphi(x',x')$ is an admissible bidding price, trader i will submit it as his own bid. On the other hand, when $\varphi(x',x')$ is greater (smaller) than the limit price, his optimal bid becomes the maximum (minimum) bid allowed by the exchange. Hence, (3.16) is optimal for trader i. Since the market clearing price $\bar{p}$ is the M^{th} highest bid, $\bar{p} = \varphi(y', y')$ where $y' = \gamma \cdot y + (1-\gamma) \cdot E[Y | X_i = y, Y \geq c]$. Notice that $\varphi(y', y') > \varphi(y, y)$ since $y' > y$. Hence, the market clearing price determined in a market with circuit breakers is greater than the one determined in a market without circuit breakers. *Q.E.D.*

Appendix 6 (A proof of lemma 2): Sophisticated trader i's maximization problem is given in (3.20). Suppose that the price functional $\pi(Y)$ is equal to $\varphi(Y, Y)$. Then, the problem for trader i degenerates into the one shown in the benchmark model without corcuit breakers. The optimal strategy $\bar{b}_s$ is equal to $\varphi(x, x)$. Next, suppose that $\pi(Y) > \varphi(Y, Y)$. Then, the optimal bid is a solution to the following:

$$\underset{\underline{\delta} \leq b \leq \bar{\delta}}{Max} \int_c^{\pi^{-1}(b)} \{\varphi(x, \omega) - \pi(\omega)\}\ h(\omega / x, \omega \geq c)\ d\omega$$

Notice that $\varphi(x, Y) - \pi(Y)$ is positive at a sufficiently small value of Y and negative at any value of Y greater than x. Since $\varphi(x, Y) - \pi(Y)$ is a monotonically decreasing function in Y, there exists a unique value of Y denoted by y' such that $\varphi(x, y') = \pi(y')$. Since $\varphi(x, Y) - \pi(Y)$ is negative when $Y=x$, y' is smaller than x. Regardless of the conditional density of Y, the maximum is achieved by integrating over Y such that $\{Y | \varphi(x, Y) - \pi(Y) \geq 0\}$. Hence, $\pi^{-1}(b) = y'$ *i.e.,* $\bar{b}_S = \pi(y')$. Since $\bar{b}_S = \pi(y')$ and $\pi(y') = \varphi(x, y') < \varphi(x, x)$, the optimal bid $\bar{b}_S$ is smaller than $\varphi(x, x)$. In other case when $\pi(Y) < \varphi(Y, Y)$, we can be prove using similar arguments. *Q.E.D.*

References

Admati, A.R. and Pfleiderer, P., A Theory of Intraday Patterns: Volume and Price Variability, *The Review of Financial Studies* **1** (1988), 3-40.

Arthur, W.B., On Learning and Adaptation in the Economy, 1992, Working Paper, Santa Fe Institute.

Bertero, E., and Mayer, C., Structure and Performance: Global Interdependence of Stock Markets Around the Crash of October 1987, *European Economic Review* **34** (1990), 1155-1180.

Bewley, T. F. *(edit),* Advances in Economic Theory: Fifth World Congress, 1987, Cambridge University Press.

Bikhchandani, S. and Riley, J. G., Equilibria in Open Common Value Auctions, *Journal of Economic Theory* **53**, (1991), 101-130.

Black, F., Noise, *The Journal of Finance* **42**, (1986), 529-543.

Bollerslev, T., A Conditionally Heteroscedastic Time Series Model for Speculative Prices and Rates of Return, *The Review of Economics and Statistics* **69** (1987) 542-547.

Brady Commission, Report of the Presidential Task Force on Market Mechanisms, submitted to the President of the United States, the Secretary of the Treasury and the Chairman of the Federal Reserve Board, 1988, U.S. Government Publications Office, Washington, D.C.

Brennan, M., A Theory of Price Limits in Future Markets, *Journal of Financial Economics* **16** (1985), 213-233.

Camerer, C. and Weigeit, K., Informational Mirages in Experimental Asset Markets, *Journal of Business* **64**, *(1991), 463-493.*

Chiang, A., Fundamental Methods of Mathematical Economics, 2nd ed., 1974, McGraw-Hill.

Conrad, J. and Kaul, G., Mean Reversion in Short-Horizon Expected Returns, *The Review of Financial Studies* **2** *(1989), 225-240.*

____________________ **and Nimalendran, M.**, Components of Short-Horizon Individual Security Returns, *Journal of Financial Economics* **29** (1991), 365-384.

De Long, J., Shleifer, A., Summers, L. and Waldman, R., Noise Trader Risk in Financial Markets, *Journal of Political Economy* **98** (1990), 703-738.

DeGroot, M.H., Optimal Statistical Decisions, 1970, McGraw-Hill.

Engle, R.F., Autogressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica* **50** (1982), 987-1007.

Fama, E.F., Efficient Capital Markets: A Review of Theory and Empirical Work, *The Journal of Finance* **25** (1970), 383-417.

Friedman, D., A Simple Testable Model of Double Auction Markets, *Journal of Economic Behavior and Organization* **15** (1991), 47-70.

____________ **and Aoki, M.**, Inefficient Information Aggregation as A Source of Asset Price Bubbles, *Bulletin of Economic Research* **44** (1992), 251-279.

Gerety, M.S., and Mulherin, J.H., Trading Halts and Market Activity: An Analysis of Volume at the Open and the Close, *Journal of Finance* **47** (1992), 1765-1784.

Greenwald, B.C. and Stein, J.C., The Task Force Report: The Reasoning Behind the Recommendations, *Journal of Political Economy* **2** (1988), 3-23.

______________________________, Transactional Risk, Market Crashes, and the Role of Circuit Breakers, *Journal of Business* **64** (1991), 443-462.

Glosten, L.R. and Milgrom, P.R., Bid, Ask and Transaction Prices in a Specialist Market with Heterogenously Informed Traders, *Journal of Financial Economics* **14** (1985), 71-100.

Grossman, S.J., Introduction to NBER Symposium on the October 1987 Crash, *Review of Financial Studies* **3** (1990), 1-3.

Hamao, Yasushi and Hasbrouck, Securities Trading in the Absence of Dealer, 1993, *Mimeo.*

Heiner, R.A., Uncertainty, Signal-Detection Experiments, and Modeling Behavior, Langlois, R.N., ed. Economics as a Process: Essays in the New Institutional Economics, Cambridge University Press, 1985, 59-115.

Huang, C. and Litzenberger, R., Foundations for Financial Economics, 1988, New York, North-Holland.

Korea Stock Exchange, Korea Stock Exchange, 1992

Kryzanowshi, L., The Efficacy of Trading Suspensions: A Regulatory Action Designed to Prevent the Exploitation of Monopoly Information, *Journal of Finance* **36** (1979), 1187-1200.

Kuhn, B. A., Kuserk, G. J. and Locke, P., Do Circuit breakers Moderate Volatility?: Evidence from October 1989, *Review of Futures Market 10* (1990), 136-179.

Kyle, A.S., Continuous Auctions and Insider Trading, *Econometrica* **53** (1985), 1315-1335.

Lee, In Ho, Market Crashes and Informational Cascades, 1992, *Mimeo.*

Leland, H. and Rubinstein, M., Comments on the Market Crash: Six Months After, *Journal of Political Economy* **2** (1988), 45-50.

LeRoy, S.F., Efficient Capital Markets and Martingales, *Journal of Economic Literature* **27** (1989), 1583-1621.

Leijonhufvud, A., High Inflations and Contemporary Monetary Theory, Working Paper No. 638, Department of Economics, University of Los Angeles, 1991.

Lo, A.W. and MacKinlay, A.C., Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, *The Review of Financial Studies* **1** (1988), 41-66.

Mann, R.P. and Sofianos, G., Circuit Breakers for Equity Markets, Appendix E of Market Volatility and Investor Confidence Panel, the New York Stock Exchange, 1990.

Market Volatility and Investor Confidence Panel, Report to the Board of Directors of the New York Stock Exchange, 1990.

McAfee, R.P., A Dominant Strategy Double Auction, *Journal of Economic Theory* **56**, (1992), *434-450.*

___________ **and McMillan, M.**, Auctions and Bidding, *Journal of Economic Literature* **25**, (1987), 699-738.

McCabe, K., Rassenti, R. and Smith, V., Designing Call Auction Institutions: Is Double Dutch the Best?, *The Economic Journal* **102**, (1992), *9-23.*

McMillan, Henry, Circuit Breakers in the S&P 500 Futures Market: Their Effect on Volatility and Price Discovery in October 1989, *Review of Futures Market* **10** (1990), 249-281.

Milgrom, P.R., A Convergence Theorem for Competitive Bidding with Differential Information, *Econometrica* **47** (1979), 679-688.

____________, Rational Expectations, Information Acquisition and Competitive Bidding, *Econometrica* **49** (1981), 921-943.

____________, Auctions and Bidding: A Primer, *Journal of Economic Perspective* **3** (1989), *3-22.*

____________ **and Stokey, N.**, Information, Trade and Common Knowledge, *Journal of Economic Theory* **26** (1982), 17-27.

____________ **and Weber, R.J.**, A Theory of Auctions and Competitive Bidding, *Econometrica* **50** (1982), 1089-1122.

Miller, M.H., Final Report of the Committee of Inquiry, Black Monday and the Future of Financial Markets, 1989, Irwin, Homewood, Illinois, 205-225.

____________, Financial Innovations and Market Volatility, 1991, Blackwell Publishers, Cambridge, Massachusetts.

Mood, Graybill and Boes, Introduction to the Theory of Statistics, 1974, McGraw-Hill.

New York Stock Exchange, Fact Book for the year 1991, 1992.

Park, J., A Study on Investors' Behavior in Presence of Price Limits *(in Korean)*, *Financial Analysis 3* (1990), 135-153.

Roll, Richard, The International Crash of October 1987, Black Monday and the Future of Financial Markets, 1989, Irwin, Homewood, Illinois, 35-70.

Sargent, T.J., Bounded Rationality in Macroeconomics, 1993.

Shiller, R.J., Market Volatility, 1989, The MIT Press, Cambridge, Massachusetts.

Shleifer, A., and Summers, L.H., The Noise Trader Approach to Finance, *Journal of Economic Perspective* **4** (1990), 19-33.

Smidt, S., Which Road to an Efficient Stock Market: Free Competition or Regulated Monopoly, *Financial Analysts Journal* (1971), 18-20 and 64-69.

Stoll, H.R., The Stock Exchange Specialist System: An Economic Analysis, Monograph Series in Finance and Economics 1985-2, Graduate School of Business Administration, New York University, 1-51.

Subrahmamyam, A., Circuit Breakers and Market Volatility: A Theoretical Perspective, *forthcoming in Journal of Finance.*

Takagi, Shinji, The Japanese Equity Market: Past and Present, *Journal of Banking and Finance* **13** (1989), 537-570.

The Economist, A Survey of the Frontiers of Finance, October 9, 1993, 1-22.

Value Line Publishing, Inc., Value Line Industry Review, 1993.